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A consideration on robust design optimization problem through formulation of multiobjective optimization

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Abstract

The robust design optimization (RDO) problem is generally formulated as a weighted sum of the nominal objective function and the robust term. In the RDO problem, a deterministic optimum design is regarded as one of the local optima. However, this property is not well understood. Even though robust optimum designs are known to be significantly different from deterministic designs in certain cases, they are nearly identical in other cases, for reasons that are not intuitively understandable. This is due to the fact that the trade-off relationship between deterministic and robust optimum designs and the effects of uncertainty on the latter are not evaluated by the weighted sum approach. In this study, the properties of robust optimum designs are investigated by formulating the RDO problem as a multiobjective optimization problem, where the nominal value of the performance function and the worst value in the uncertainty region are adopted as the objective functions. The problem considered in this study is limited in that for simplicity, only the design variable is assumed to have uncertainty. That is, the mean value of the random variable is regarded as the design variable. The Pareto solutions are obtained by an evolutionary algorithm whereby the worst design in each individual during the evolutionary process is selected by a sampling method so that the approximation error may be avoided. Through simple numerical examples under several distribution types for random variables, the trade-off relationship between deterministic and robust optimum designs and the effects of uncertainty on the latter are investigated.

Key words : Robust design, Multiobjective optimization, Uncertainty, Pareto set, Trade-off

1. Introduction

Robust design optimization (RDO) is one of reasonable design approaches considering uncertainty. An outline of its history and methodology can be found in several comprehensive review studies [Ben-Tal and Nemirovski, 2002, Beyer and Sendhoff, 2007, Park et al., 2006, Schuëller and Jensen, 2008]. In RDO, a design insensitive to variations of the design parameters is called robust. For example, it is generally said that point B in Fig. 1 is more robust than point A because the worst value of the objective function by perturbation in the tolerance range at A is significantly larger than at point B. When the uncertainty range is parametrically changed, the robust design is selected between A and B. However, A will be selected as the robust design, when the tolerance range is smaller, because the worst design under perturbation at A is better than at B.

It is easily found that a deterministic optimum design is one of local optima in the RDO problem. However, this property is not clearly understood. This is due to the fact that the tradeoff relationship and the effects of uncertainty on the robust optimum design are not evaluated by the weighted sum approach. For example, it has been observed that the robust topology optimum design is quite similar to the deterministic optimum design in a certain load case [Nakazawa et al., 2016]. However, as the study used a weighted sum approach, the effect of uncertainty was not clarified in detail.

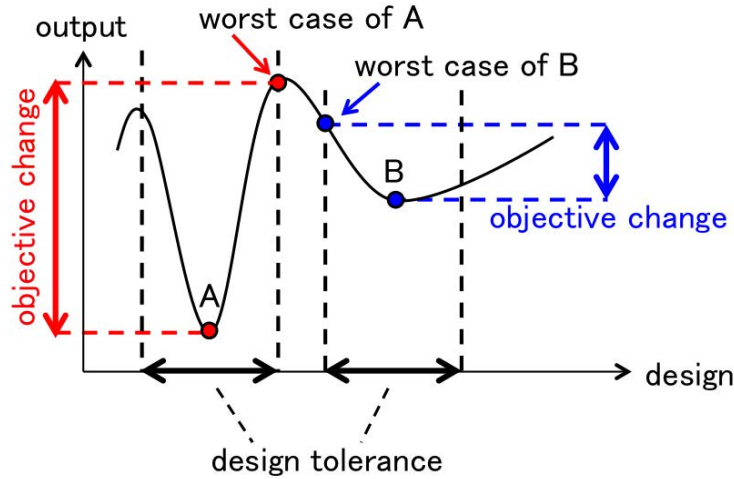


Fig. 1 General definition of robust design on the objective function. Generally, the point B is said to be more robust than the point A, because the deterioration of the objective function value in the design tolerance at B is smaller than at A.

In this study, the effect of uncertainty on robustness is investigated by formulating the RDO problem as a multiobjective optimization problem, where the nominal value of the performance function and the worst value in the uncertainty region are adopted as the other objective functions [Toyoda and Kogiso, 2015]. In this problem, the design variable is assumed to have uncertainty. That is, the mean value of the random variable is regarded as the design variable.

As search for the worst design in the uncertainty range is a time-consuming process, the linear approximation method has been widely adopted [Chen et al., 1996, Kannno and Takewaki, 2007, Lee et al., 2002, Takezawa et al., 2011], whereby the nominal value of the objective function is represented as the mean of the objective function, and deterioration is represented as the variance of the objective function in terms of random variables. Their values are usually obtained by the following linear approximations:

$$\text{Mean: } E[f(\mathbf{x})] \approx f(E[\mathbf{x}]) = f(\boldsymbol{\mu}_x) \tag{1}$$

$$\text{Variance: } \text{Var}[f(\mathbf{x})] \approx \sum_{i=1}^{n_d} \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2 + \sum_{i=1}^{n_d} \sum_{\substack{j=1 \\ i \neq j}}^{n_d} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \text{Cov}(x_i, x_j) \tag{2}$$

where n_d is the number of random variables, and $\boldsymbol{\mu}_x$ is the mean value of the random variable, that is,

$$\boldsymbol{\mu}_x = (E(x_1), E(x_2), \dots, E(x_{n_d})) = (\mu_{x_1}, \mu_{x_2}, \dots, \mu_{x_{n_d}}). \tag{3}$$

σ_{x_i} and $\text{Cov}(x_i, x_j)$ are the standard deviation of the random variable x_i and the covariance between x_i and x_j , respectively. They are given by the following covariance matrix Σ .

$$\Sigma = \begin{pmatrix} \sigma_{x_1}^2 & \text{Cov}(x_1, x_2) & \cdots & \text{Cov}(x_1, x_{n_d}) \\ \text{Cov}(x_2, x_1) & \sigma_{x_2}^2 & \cdots & \text{Cov}(x_2, x_{n_d}) \\ \vdots & \cdots & \ddots & \vdots \\ \text{Cov}(x_{n_d}, x_1) & \text{Cov}(x_{n_d}, x_2) & \cdots & \sigma_{x_{n_d}}^2 \end{pmatrix} \tag{4}$$

When the random variables are independent, the covariance is eliminated; hence, the covariance matrix is diagonal.

In the robust multiobjective optimization problem, the trade-off between the nominal and the worst values of the objective function can be visualized by the Pareto frontier. In this study, the Pareto solution is evaluated by the multiobjective evolutionary algorithm (MOEA) [Deb, 2001, Jin and Sendhoff, 2003]. As simple mathematical problems are investigated, the computational cost is not considered. In addition, the search for the worst design in the given uncertainty region is performed by a sampling method to avoid approximation error in Eqs. (1) and (2) [Arakawa and Yamakawa, 2016]. Even though this process is time-consuming, the worst design can be evaluated accurately regardless of the probability distribution and correlation between the random variables.

In this study, the ill-understood property of the RDO problem that a deterministic optimum design is one of the local optima will be clarified by formulating the RDO problem as a multiobjective optimization problem. For this purpose, the

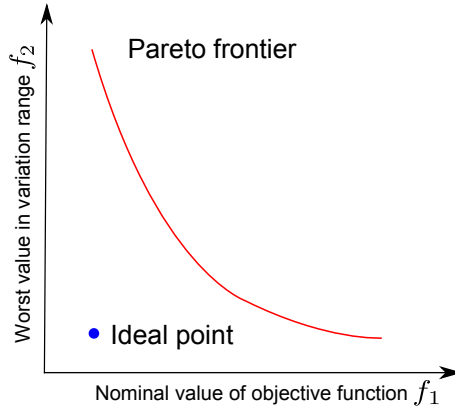


Fig. 2 Trade-off relationship between the nominal and the worst values.

trade-off relationship between the deterministic and robust optimum designs will be visualized by investigating the Pareto frontier under several random variable settings in simple numerical examples.

This paper is organized as follows. In Section 2, the RDO framework is described using evolutionary algorithms. In Section 3, the property of robust designs is discussed using two numerical examples under several conditions. Section 4 concludes the paper.

2. Robust Design Framework

2.1. Formulation of Robust Design Optimization (RDO)

In this study, it is assumed that the design variable \mathbf{d} itself has variation, which is described as a random variable \mathbf{x} . That is, the mean of the random variable \mathbf{x} is set as a design variable \mathbf{d} . The variation region S of the random variable is limited within the standard deviation around the mean value \mathbf{x} as follows:

$$S = \{ \mathbf{x} \mid (\mathbf{x} - \mathbf{d})^T \Sigma^{-1} (\mathbf{x} - \mathbf{d}) \leq R^2 \} \tag{5}$$

where Σ is the covariance matrix, and R is set so that the variation range corresponds to $\mu \pm \sigma$ for the independent random variable.

The worst value of the objective function $f^w(\cdot)$ in the variation region is defined as follows:

$$f^w(\mathbf{d}) = \max_{\mathbf{x} \in S} [f(\mathbf{x})] \tag{6}$$

Then, the robust optimization problem is formulated as the following multiobjective optimization:

$$\text{Minimize: } f_1(\mathbf{d}) = f(\mathbf{d}) \tag{7}$$

$$f_2(\mathbf{d}) = |f^w(\mathbf{d}) - f(\mathbf{d})| \tag{8}$$

The first objective function $f_1(\cdot)$ represents the minimization of the nominal value of the original objective function $f(\cdot)$. $f_2(\cdot)$ represents robustness, where the difference between the nominal value and the worst value of the objective function in the variation region is minimized. Using this formulation, the trade-off between the nominal value and the worst value of the objective function can be investigated.

When the optimization problem has constraints, the formulation is adjusted by adopting the exterior penalty function method. That is, the first objective function, Eq. (7), is transformed to the following penalty function [Kitayama et al., 2005]:

$$f_1(\mathbf{d}) = f(\mathbf{d}) + r \sum_{j=1}^m p_j(\mathbf{d}) \tag{9}$$

where r is a penalty factor and p_j is a penalty function for the j -th constraint function ($g_j(\mathbf{d}) \leq 0$) defined as follows:

$$r = (1 + |f(\mathbf{d})|)^2 \tag{10}$$

$$p_j(\mathbf{d}) = \begin{cases} \exp(1 + |g_j(\mathbf{d})|) & \text{if } g_j(\mathbf{d}) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

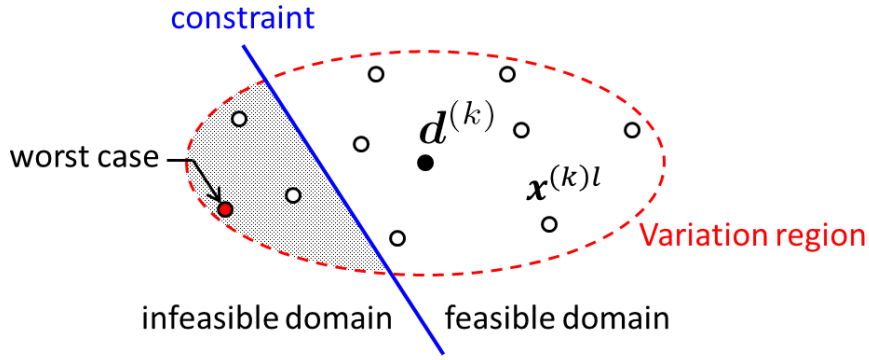


Fig. 3 Sampling for the worst case evaluation. n_s samples are generated around the nominal design $\mathbf{d}^{(k)}$ within the variation region. Then, the worst case is selected to evaluate robustness.

Then, the second objective function Eq. (6) is also converted to a penalty function as follows:

$$f^w(\mathbf{d}) = \max_{\mathbf{x} \in \mathcal{S}} [f_1(\mathbf{x})] \tag{12}$$

It should be noted that the definition of f_2 is not changed. That is, the nominal value of the objective function is not penalized, so that the difference between the nominal and the worst values will be larger when the nominal design is violated.

NSGA-II [Deb, 2001] is adopted as general MOEA for evaluating a Pareto set for RDO to investigate the effect of variations on the Pareto solution distribution.

2.2. Worst case analysis

In this study, the worst design for the k -th individual design $\mathbf{d}^{(k)}$ is selected from samples distributed in the uncertainty region to avoid approximation error. That is, the worst design $f^w(\mathbf{d}^{(k)})$ in Eq. (12) is redefined as follows:

$$f^w(\mathbf{d}^{(k)}) = \max_{1, \dots, n_s} [f_1(\mathbf{x}^{(k)1}), \dots, f_1(\mathbf{x}^{(k)n_s})] \tag{13}$$

where n_s is the number of samples and $\mathbf{x}^{(k)l}$ is the l -th sample, as shown in Fig. 3. It should be noted that the sample distributed in the infeasible domain is penalized, as defined in Eq. (9) and shown in Fig. 3.

Then, the second objective function corresponding to the worst case can be evaluated from these samples as

$$f_2(\mathbf{d}^{(k)}) = |f^w(\mathbf{d}^{(k)}) - f(\mathbf{d}^{(k)})| \tag{14}$$

where the nominal design is not penalized even when $\mathbf{d}^{(k)}$ lies in the infeasible region to make the difference between the nominal and the worst design larger. As mentioned above, the first objective function is penalized when $\mathbf{d}^{(k)}$ lies in the infeasible region.

2.3. Implementation of RDO

A general MOEA is adopted for evaluating a Pareto set for RDO to investigate the effect of variations on the Pareto solution distribution. The following optimization algorithm is adopted.

- Step 1** Set the MOEA parameters, i.e., population size and the number of generations as well as the worst case evaluation parameters, i.e., the sample size n_s , the mean $\mathbf{d}^{(k)}$, and the covariance matrix Σ .
- Step 2** n_s samples are generated for the k -th individual $\mathbf{d}^{(k)}$.
- Step 3** The first objective function $f_1(\cdot)$ in Eq. (9) is evaluated for the k -th individual $\mathbf{d}^{(k)}$.
- Step 4** The second objective function $f_2(\cdot)$ in Eq. (14) is evaluated for the k -th individual followed by the worst case evaluation in Eq. (13) from n_s samples.
- Step 5** If $k = N_{\text{pop}}$, terminate the evaluation of fitness in current generation. Otherwise, update as $k = k + 1$ and return to Step 2.
- Step 6** The Pareto frontier is updated through the MOEA process.
- Step 7** If the number of generations reaches the stopping criterion, the search is finished. Otherwise, return to Step 2 for the next generation.

Table 1 Types of random variables in example 1

	Distribution type	Standard deviation
Case 1	Normal	0.1
Case 2	Normal	0.05
Case 3	Normal	0.3
Case 4	Uniform	0.1
Case 5	Log-normal	0.1
Case 6	Gumbel	0.1
Case 7	Uniform	0.3
Case 8	Log-normal	0.3
Case 9	Gumbel	0.3

3. Discussion of Robustness through Numerical Examples

In this section, two numerical examples are presented to discuss the property of robust designs. NSGA-II [Deb et al., 2002], one of the best known MOEA methods, is adopted for solving the proposed robust multiobjective optimization problem.

3.1. Numerical example 1: one-dimensional problem

The first example is a simple one-dimensional unconstrained problem. The original optimization problem with only a side constraint is described as follows:

$$\begin{aligned}
 &\text{Find } d(= d_1) && (15) \\
 &\text{minimize: } f(x) = (x_1 - 3)^4 - 3(x_1 - 3)^2 + 4(x_1 - 3) \\
 &\text{subject to: } 0 \leq d_1 \leq 6
 \end{aligned}$$

where d_1 is a design variable and is the mean value of the random variable x_1 .

As the adopted method can treat any probability distribution, several types, described in Table 1, are applied. Specifically, cases 1–3 correspond to the normal distribution with different standard deviation, whereas cases 4 to 6 and cases 7 to 9 correspond to different distributions with the same standard deviation as in case 1 and case 3, respectively. The size of the population and the number of generations in NSGA-II are set to 50 and 200, respectively, and the number of samples for the worst design searching in individual region is set to 100.

The obtained Pareto solutions are shown in Figs. 4, 5 and 6, where the left hand side denoted as (i) corresponds to the Pareto solutions as red and purple marks on the objective function curve and the right hand side, (ii), indicates the Pareto curve in the objective function space. The mark color is used to emphasize on the separation of the Pareto curve both in design variable and objective function spaces.

The results in the case where the random variable follows normal distribution in Fig. 4 are first compared. As shown in their Pareto frontiers in the design variable space, the range of the Pareto curve is affected by standard deviation. For smaller standard deviation in cases 1 and 2, the Pareto frontier is separated to the two parts closer to the global optimal point ($d_1 = 1.5$) and the deflection point $d_1 = 3.7$, where the slopes of the objective function with respect to the design variable nearly vanish. By contrast, the Pareto curve is nearly connected for the largest standard deviation in case 3, where there exist balanced designs between deterministic and robust optimum designs.

Subsequently, the results of different probability distributions with the same standard deviation are compared in Figs. 5 and 6. It is seen that the Pareto curve under log-normal distribution in Fig. 5 (b) is similar to that under normal distribution in Fig. 4 (a). However, for larger standard deviation, the Pareto frontier for left-hand side is different such that the Pareto solutions in log-normal distribution are concentrated to bottom point in design variable space as shown in Fig. 6 (b). In addition, the Pareto solutions under uniform distribution (case 4) consists of two narrow regions, as shown in Fig. 5 (a). For larger standard deviation (case 7), Pareto solutions are separated as shown in Fig. 6 (a). By contrast, the Pareto solutions under Gumbel distribution (case 6) are distributed primarily around $d_1 = 3.1$, as shown in Fig. 5 (c). For larger standard deviation shown in Fig. 6 (c), the Pareto solutions are shifted to the smaller d_1 region and almost connected to the left part.

The distributions with the same mean value ($\mu = 3.0$) and standard deviation ($\sigma = 0.1$ and $\sigma = 0.3$ described in Table 1) are compared in Fig. 7. It can be seen that the log-normal distribution is almost identical to the normal distribution, and the Gumbel distribution has the largest variation on the left side, whereas the uniform distribution has the smallest

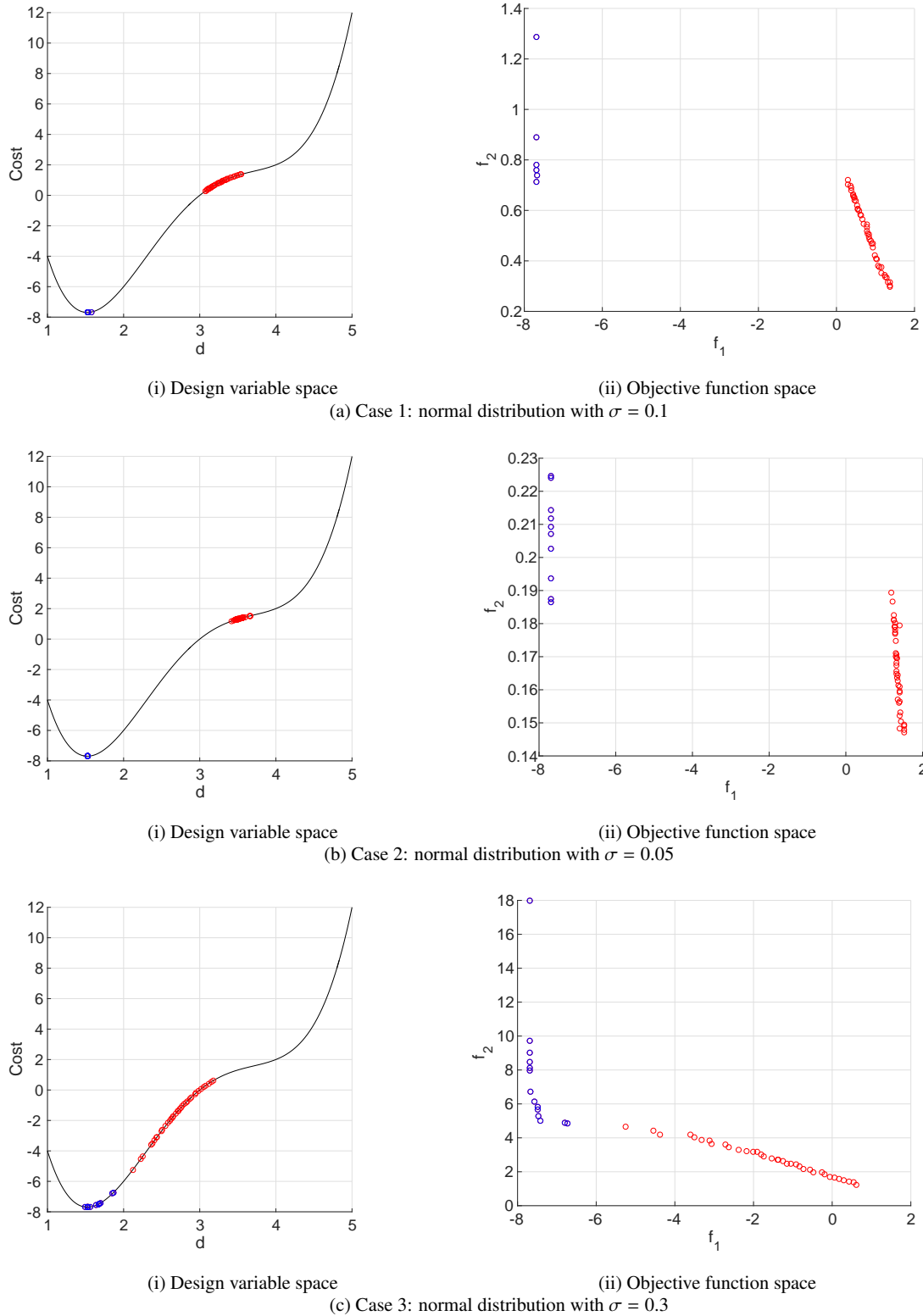


Fig. 4 Pareto frontier for different standard deviations in normal distribution in example 1.

variation. Therefore, the Pareto frontier for the Gumbel distribution with $\sigma = 0.3$ forms continuous curve as shown in Fig. 6 (c). The property under investigation of the Pareto solutions is shown to depend on the distribution type.

3.2. Comparison with weighted-sum approach

In this study, to investigate the property of robust designs, multiobjective optimization is adopted. The weighted sum method is widely used, in which the multiobjective optimization problem is converted into a single objective optimization

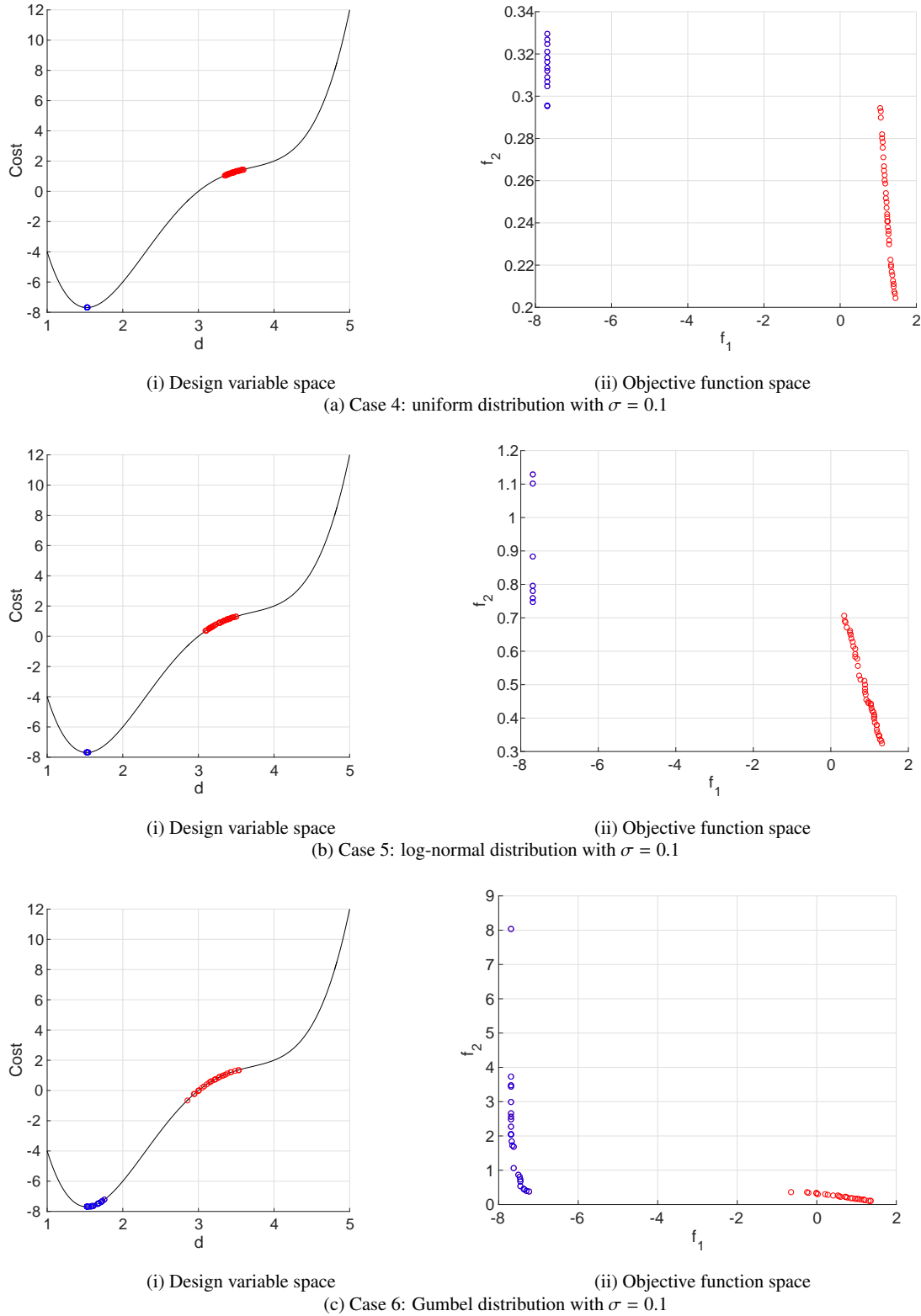


Fig. 5 Pareto frontier for different distribution types with $\sigma = 0.1$ in example 1.

problem as follows:

$$\text{Minimize: } f_{\text{conv}}(\mathbf{x}) = w f_1(\mathbf{x}) + (1 - w) f_2(\mathbf{x}) \tag{16}$$

where w is a weighting factor ($0 \leq w \leq 1$) whereby the balanced solution is obtained.

However, the weighted-sum approach is known to have the problem that the concave Pareto solution in the objective function space is not obtained even if the weighting factor is parametrically changed [Nakayma and Sawaragi, 1984]. This

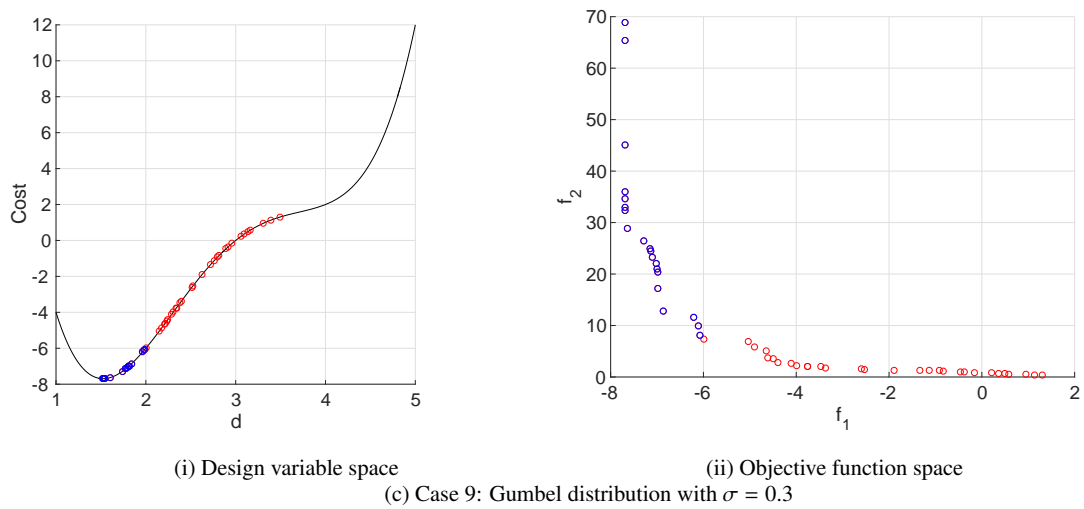
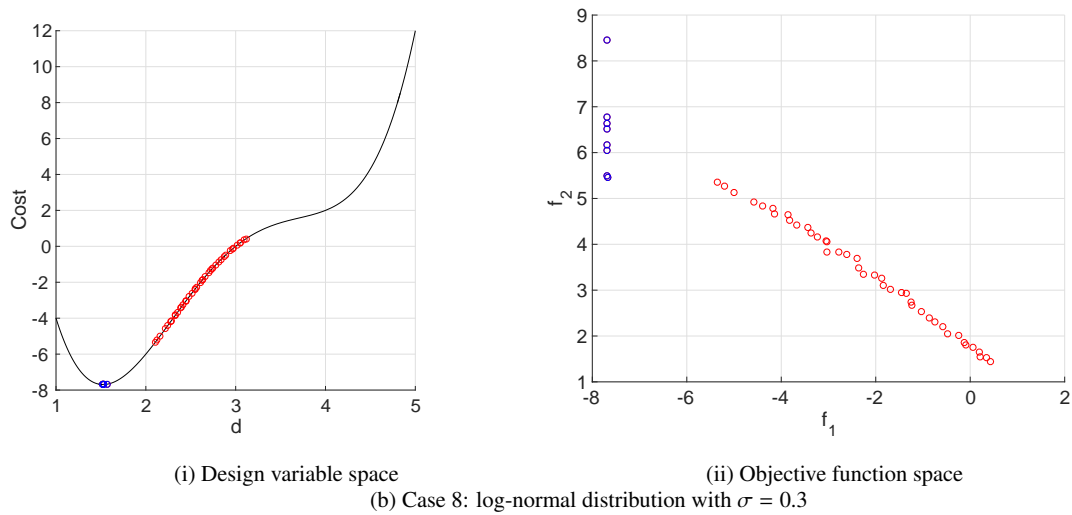
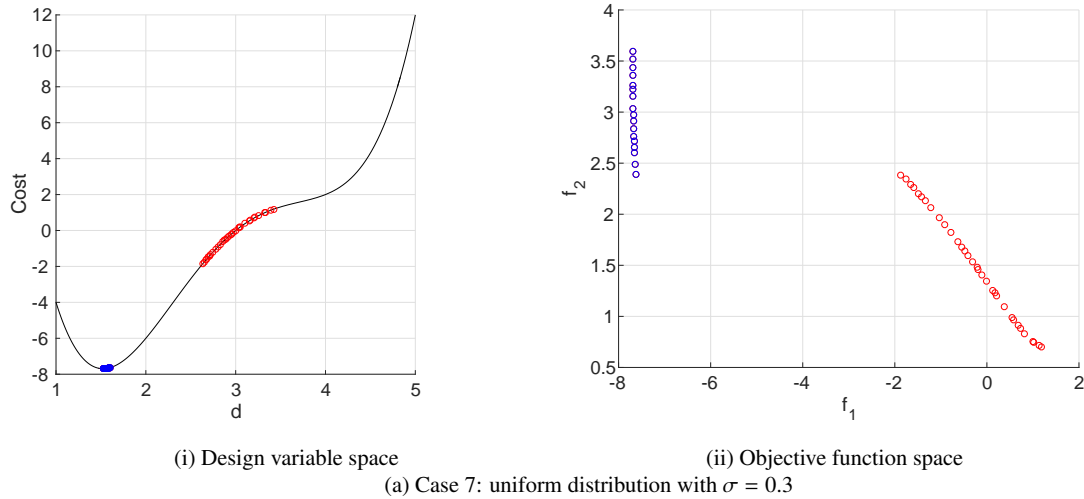
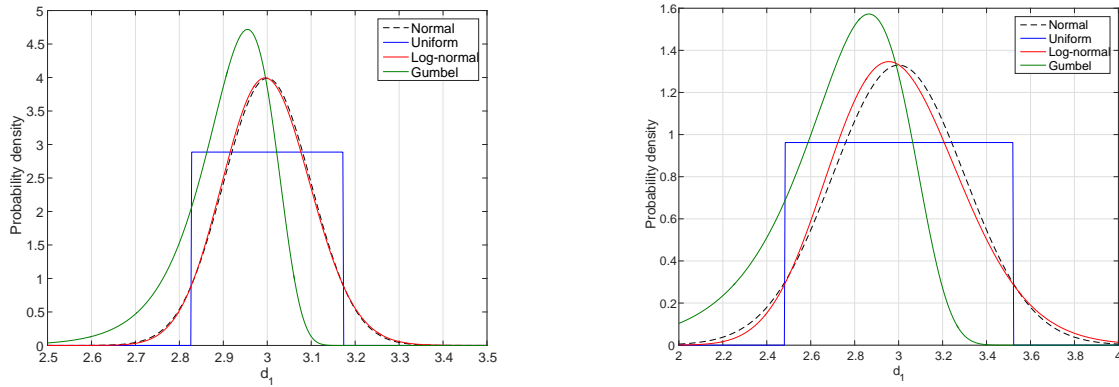


Fig. 6 Pareto frontier for different distribution types with $\sigma = 0.3$ in example 1.

implies that the concave part of the Pareto curves in Figs. 4–5 (b) for example 1 are difficult to obtain. Even if the weight is put on the robust term, the deterministic design will be obtained, except for the case where w is nearly equal to zero.

3.3. Balance between deterministic and robust optimum designs

In this example, the robust designs located in the rightmost region in the objective function space shown in Figs. 4, 5



(a) $\sigma = 0.1$ in Case 1: normal distribution, case 4: uniform distribution, case 5: log-normal distribution, and case 6: Gumbel distribution (b) $\sigma = 0.3$ in Case 3: normal distribution, case 7: uniform distribution, case 8: log-normal distribution, and case 9: Gumbel distribution

Fig. 7 Comparison of probability density functions in example 1.

and 6 have considerably larger value of the nominal objective function $f_1(\mathbf{d})$ than the deterministic design in the leftmost region in the same figures.

Such robust designs are not preferred as candidates in general design problems because the nominal objective function value is considerably larger than the deterministic. Except for cases 3 and 9, where the Pareto solution is not separated, no balanced design can be obtained between the nominal and the robust designs, and the deterministic optimum design is usually selected, even if uncertainty is considered. In this case, a “robust” design may not be required. However, this does not imply that considering uncertainty is not important; rather, the robust design happens to be the same as the deterministic design.

In cases 3 and 9, the balanced design with the largest robustness and smallest nominal value may be selected for the objective function. It should be noted that the balanced design is not located far from the deterministic design. Moreover, as mentioned earlier, the balanced designs are not selected by the weighted-sum approach for this concave Pareto solution.

3.4. Numerical example 2: two-dimensional problem

The next example is the following two-dimensional design problem with nonlinear objective function and three nonlinear constraint functions:

$$\begin{aligned} \text{Minimize: } & f(\mathbf{x}) = -(x_1 - 1)^2 - (x_2 - 0.5)^2 & (17) \\ \text{subject to: } & g_1(\mathbf{x}) = [(x_1 - 3)^2 + (x_2 + 2)^2] \exp(-x_2^7) - 12 \leq 0 \\ & g_2(\mathbf{x}) = 10x_1 + x_2 - 7 \leq 0 \\ & g_3(\mathbf{x}) = (x_1 - 0.5)^2 - (x_2 - 0.5)^2 - 0.2 \leq 0 \\ & 0 \leq d_i \leq 1 \quad i = 1, 2 \end{aligned}$$

This is originally formulated as a deterministic design problem and is known to have two separated feasible regions. Each region has a local optimum design, and one of them is a global optimum. Thus, this problem is used as a benchmark problem [Kitayama and Yamazaki, 2014]. In this study, it is converted to an RDO problem considering variation in design variables, where the mean of the random variable x_i is set as the design variable d_i .

Two types of random variables, namely, independent and correlated variables, are considered. The covariance matrix for each case is as follows:

$$\Sigma = \begin{cases} \begin{pmatrix} 0.03^2 & 0 \\ 0 & 0.05^2 \end{pmatrix} & \text{(Case 1)} \\ \begin{pmatrix} 0.03^2 & -0.001 \\ -0.001 & 0.05^2 \end{pmatrix} & \text{(Case 2)} \end{cases} \quad (18)$$

In both cases, the variables have the same variance. However, the random variables in case 2 are correlated, with correlation coefficient $-2/3$.

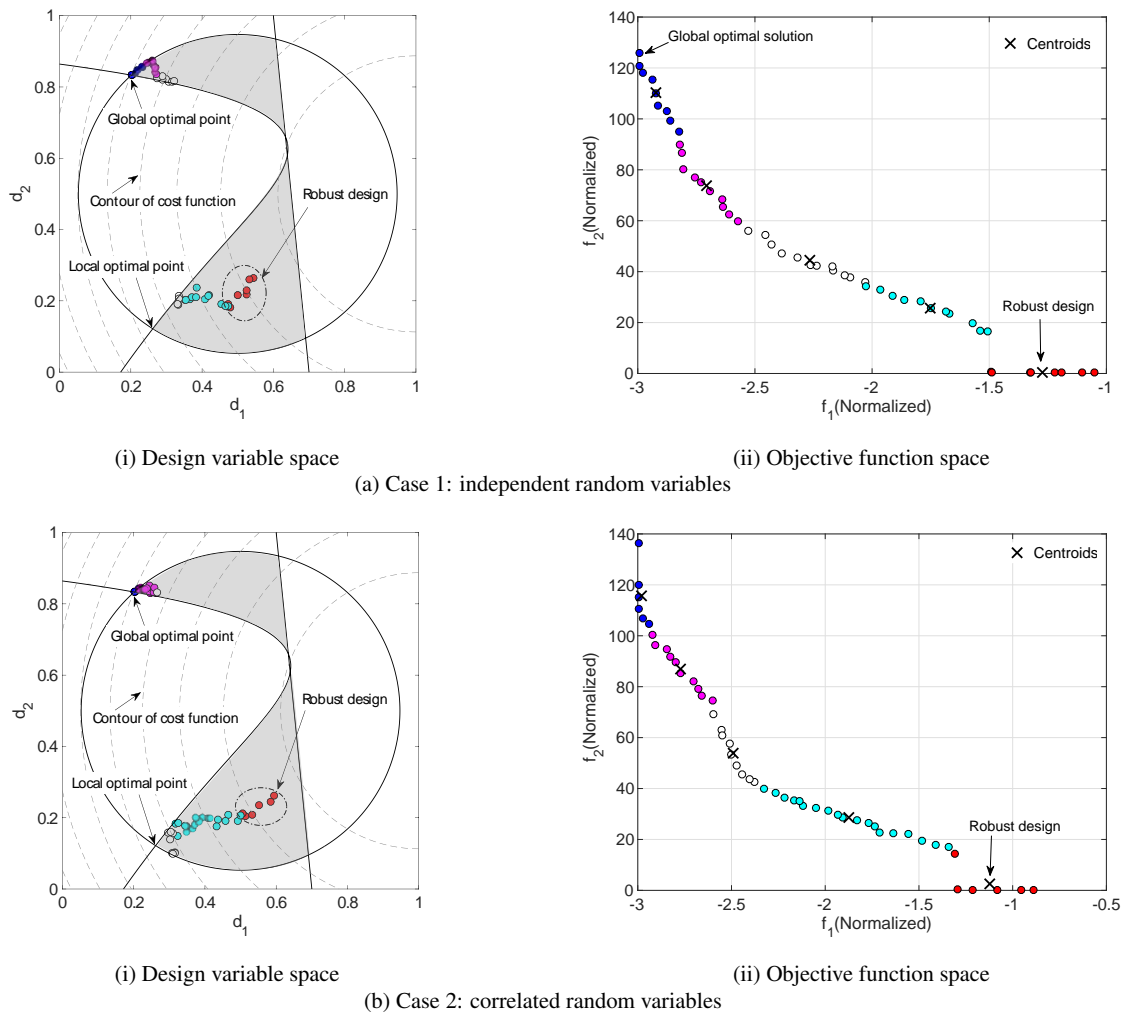


Fig. 8 Pareto frontier of robust design in example 2

The size of the population and the number of generations in NSGA-II are set to 50 and 500, respectively, and the number of samples for the worst case searching is set to 1000.

The Pareto frontiers for both cases are shown in Fig. 8 (a) and (b), where the feasible regions are hatched and the objective function values are normalized by their value at $\mathbf{d} = [1, 1]^T$ to improve visualization. The left hand side indicates the Pareto solution distribution in the design variable space, and the right hand side shows the Pareto curve in the objective function space. It can be seen that k-means clustering [MacQueen, 1967] is performed with five clusters by different colors. The deterministic optimum and the neighborhood designs, marked with blue points, are located on the upper side of the Pareto solutions in the design variable space that corresponds to the global deterministic optimum solution. The balanced designs with improved and deteriorated nominal robustness are shifted to the lower side of the Pareto solutions in the design variable space that corresponds to the local optimum point. The designs are marked with white marks. The robust designs with the largest nominal values, marked with red marks, are in the lower feasible region.

The correlated random variable appears to have a different effect on the Pareto set from the independent random variable. For the correlated random variable (case 2), the robust-side designs with red marks, which are located on the rightmost side, have a larger value of d_1 , than for the independent variable (case 1). The Pareto design that shifted from the upper feasible region (global) to the lower feasible region (local) with increased robustness has a smaller value of the nominal objective function for the correlated variable than for the independent variable.

3.5. Gap between robust and deterministic optimum designs

In both examples, there exist gaps between the deterministic and the robust designs. In the first example, the Pareto frontier has a large gap in the objective function space. This is not the case in the second example, where there is a large gap in the design variable space. This may be preferable for demonstrating robustness.

However, such a gap may be undesirable for practical design application because the selected optimum design will be considerably different under small changes of the uncertainty condition. This is similar to a deterministic optimization problem with multiple local optima. The difference is that in robust design problems, the insensitive region should be considered with respect to uncertainty related to local robust minima in addition to the local optima of the objective function.

This difference makes the RDO problem an ill-defined problem. The issue is that the difference is not visible, except for the Pareto frontier. The multiobjective optimization formulation plays an important role in the visualization of the robust design distribution and the effect of uncertainty on the objective function.

4. Conclusions

In this study, the property of robust optimum designs in the worst case scenario was visualized. The RDO problem was formulated as a multiobjective optimization problem where the nominal value of the objective function was adopted as the first objective, and the difference between the worst value in the variation range and the nominal value was adopted as the second objective function. The design variable was assumed to have variations and was set to the mean value of the random variable. The worst case was evaluated through a sampling method to avoid the approximation error incurred by conventional approaches whereby the evaluation is performed by first-order approximation of the mean and the variance. The evolutionary multiobjective optimization approach was adopted to obtain the Pareto set, as a numerical implementation.

The robust optimum design taking into account uncertainty was identical to the deterministic design in certain cases and considerably different in other cases. The possible reasons were investigated by visualizing the trade-off between the deterministic and the robust optimum design.

Using simple numerical examples, the trade-off relationship between the deterministic and the robust optimum design, shown in the Pareto frontier, was demonstrated to be affected by the uncertainty range of the random variables. The following properties were pointed out by visualizing the trade-off relationship in the Pareto frontier.

- In the first example, the Pareto frontier was separated into the deterministic-side and the robust-side solutions. In addition, the frontier had a concave shape.
- The second example with separated feasible region showed that the Pareto frontier in the design variable space had a gap, even though the Pareto solution was continuously distributed in the objective function space.
- In addition, evaluation of the worst design by sampling was applied to investigate the effect of the properties of the random variable (probability distribution) or the correlation on the Pareto set.

The importance of robust multiobjective optimization formulation was demonstrated in pointing out the effect of uncertainty on the RDO problem. However, it is difficult to directly apply the multiobjective formulation to practical design problems. Development of an efficient and applicable multiobjective optimization approach suitable to the RDO problem remains future work.

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