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著者	Toyoda Masahiro, Kogiso Nozomu
journal or publication title	Journal of Mechanical Science and Technology
volume	29
number	4
page range	1361-1367
year	2015-04-14
権利	The final publication is available at link.springer.com via http://dx.doi.org/10.1007/s12206-015-0305-9.
URL	http://hdl.handle.net/10466/15650

doi: 10.1007/s12206-015-0305-9

Journal of Mechanical Science and Technology 00 (2013) 0000~0000
www.springerlink.com/content/1738-494x

Robust multiobjective optimization method using satisficing trade-off method

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(Manuscript Received 000 0, 2013; Revised 000 0, 2013; Accepted 000 0, 2013)

Abstract

This study proposes a robust multiobjective optimization approach using the satisficing tradeoff method (STOM). STOM is a multi-objective optimization method that obtains a highly accurate single Pareto solution. Conventionally, a robust design is formulated as a single-objective optimization problem, where the objective function is defined as the weighted sum of the mean and standard deviation of the performance index. In this study, the mean and standard deviation are formulated as individual objective function. The effect of uncertainty can be investigated through Pareto surface. In STOM, the multiobjective optimization problem is transformed into the equivalent single objective problem by introducing an aspiration level. As the obtained single Pareto solution corresponds to the aspiration level that implies the ratio of the designer's desired objective function, the designer can investigate only the desired space in detail through setting the aspiration without obtaining full Pareto surfaces. The validity of using STOM for a robust multiobjective optimization problem is discussed using numerical examples.

Keywords: Multiobjective optimization; Robust design; Satisficing trade-off method; Uncertainty

1. Introduction

Robust optimal design is widely applied to engineering design problems that consider the uncertainties of design variables and parameters such as the material constants and applied load conditions [1, 2]. Robust design optimization is conventionally defined as a single-objective optimization method, where the objective function is defined as the weighted sum of the expected value and the standard deviation of the performance index. Such a weighted sum approach is one of the methods for transforming a multiobjective optimization problem into the equivalent single-objective optimization problem. It is also known that this approach sometimes fails to obtain the desired optimum solution, especially when the Pareto set has a concave shape.

This paper proposes a method for directly formulating a robust design problem as a multiobjective optimization problem [3], where both the expected value and standard deviation of the performance index are adopted as objective functions. Then, the robust design problem is solved using the satisficing trade-off method (STOM) [4]. STOM is known to be an interactive optimization method and has been applied to many different kinds of multiobjective optimization problems [5, 6]. STOM converts a multiobjective optimization problem into the equivalent single-objective optimization problem by introducing an aspiration level that corresponds to the user's preference for each objective function value. In addition, the user can interactively repeat the optimization until the desired Pareto solution is found by updating the aspiration level. The automatic trade-off analysis method [7] is one of the methods used to reasonably update the aspiration level.

The validity of using STOM for a robust multiobjective optimization problem is discussed through numerical examples. In particular, the accuracy of the Pareto set obtained by parametrically changing the aspiration level is demonstrated. Then, the effect of the random variables on the Pareto set is investigated in the small region of the Pareto set.

2. Robust Multiobjective Optimal Design

The robust design optimization is formulated to obtain the design with the smallest deterioration in performance under a variety of uncertain design parameters, as well as a reasonable higher performance, as illustrated in Fig. 1, where $\boldsymbol{z}_0$ and $\Delta\boldsymbol{z}$ denote the mean and variation of the random variables, respectively. The deterministic optimal solution $\boldsymbol{x}_{\mathrm{opt}}$ may typically show larger variation in the objective function Δf_{opt} under variation in the random variables $\boldsymbol{z}$. In contrast, the robust optimal solution will yield a smaller corresponding variation $\Delta f_{\mathrm{robust}}$ even if the objective function of the robust design $f(\boldsymbol{x}_{\mathrm{robust}})$ is worse than that of the deterministic optimal design $f(\boldsymbol{x}_{\mathrm{opt}})$.

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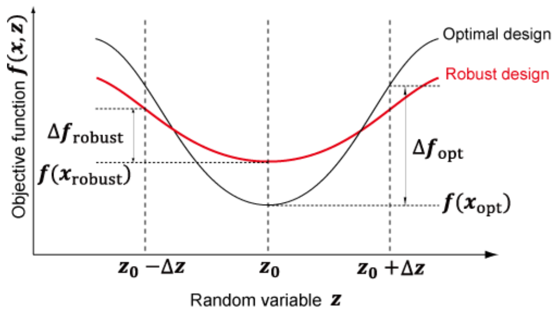


Fig. 1. Concept of robust optimization

The objective function of the robust design optimization is conventionally formulated as the weighted sum of the mean value and the standard deviation of the objective function, as follows:

$$\text{Minimize:} \quad f_r(\boldsymbol{x}) = E[f(\boldsymbol{x},\boldsymbol{z})] + \alpha\sqrt{\text{Var}[f(\boldsymbol{x},\boldsymbol{z})]} \tag{1}$$

where $\boldsymbol{x}$ and $\boldsymbol{z}$ are the design and random variables, respectively, and α is a weighting coefficient. As the linear approximation, the mean value and variance of the objective function f can be evaluated according to the following equation [8]:

$$E[f(\boldsymbol{x},\boldsymbol{z})] \approx f(E[\boldsymbol{x}], E[\boldsymbol{z}]) \tag{2}$$

$$\text{Var}[f(\boldsymbol{x},\boldsymbol{z})] \approx \sum_{i=1}^{n_x}\left(\frac{\partial f}{\partial x_i}\right)^2 \text{Var}[x_i] + \sum_{i=1}^{n_z}\left(\frac{\partial f}{\partial z_i}\right)^2 \text{Var}[z_i] \tag{3}$$

where n_x and n_z are the numbers of the design and random variables, respectively.

The single-objective optimization using Eq. (1) sometimes fails to obtain the desired solution, especially when the Pareto set has a non-convex shape, as shown in Fig. 2. In this study, the robust design optimization is formulated as the following multiobjective optimization problem:

$$\begin{aligned} \text{Minimize:} \quad & \boldsymbol{f}(\boldsymbol{x},\boldsymbol{z}) = (f_1, f_2) \\ & f_1 = E[f(\boldsymbol{x},\boldsymbol{z})] \\ & f_2 = \sqrt{\text{Var}[f(\boldsymbol{x},\boldsymbol{z})]} \\ \text{subject to:} \quad & g_j(\boldsymbol{x},\boldsymbol{z}) \le 0 \quad (j = 1,\cdots,m) \\ & x_i^L \le x_i \le x_i^U \quad (i = 1,\cdots,n_x) \end{aligned} \tag{4}$$

where $g_j(\boldsymbol{x}, \boldsymbol{z})$ and $(j = 1, ..., m)$ are constraint conditions, and x_i^U and x_i^L are the upper and lower limits of the design variables, respectively.

3. Satisficing Trade-off Method (STOM)

STOM is known to be an interactive optimization method and converts a multiobjective optimization problem into the equivalent single-objective optimization problem by introducing an aspiration level that corresponds to the user's preference for each objective function value. The flow of the STOM is summarized in Fig. 3 and briefly described as follows.

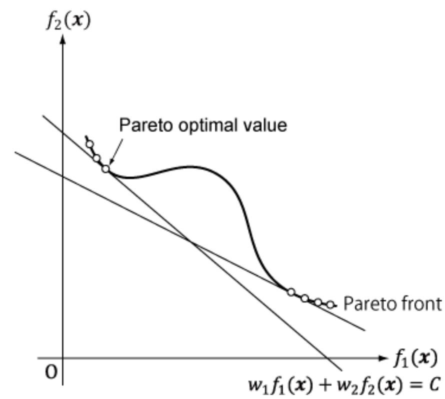


Fig. 2. Case of non-convex Pareto set

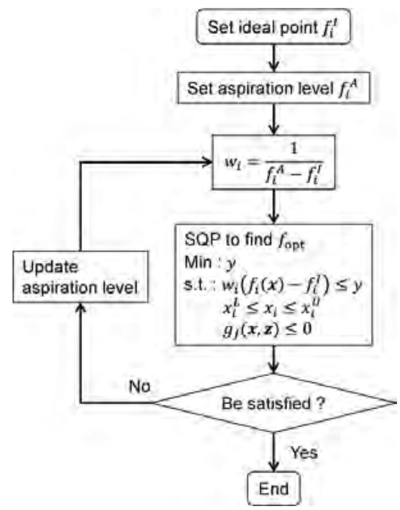


Fig. 3. Computational flow of STOM

Step 1: Set the ideal point f_i^I, $(i = 1, ..., k)$ of each objective function. The ideal point is usually determined by solving a single-objective optimization problem considering only the corresponding objective function, f_i $(\boldsymbol{x}, \boldsymbol{z})$. The ideal point for the mean performance is obtained by solving the deterministic design problem.

Step 2: Set the aspiration level f_i^A, $(i = 1, ..., k)$ of each objective function and evaluate the weight coefficient as follows:

$$w_i = \frac{1}{f_i^A - f_i^I}, \quad (i = 1,\cdots,k) \tag{5}$$

Step 3: Formulate the multiobjective optimization problem in Eq. (4) into the weighted Tchebyshev norm problem as follows:

$$\begin{aligned} \text{Minimize:} \quad & \max_{i=1,\cdots,k} w_i\left(f_i(\boldsymbol{x}) - f_i^I\right) \\ \text{subject to:} \quad & g_j(\boldsymbol{x},\boldsymbol{z}) < 0, \quad (j = 1,\cdots,m) \\ & x_i^L \le x_i \le x_i^U \quad (i = 1,\cdots,n_x) \end{aligned} \tag{6}$$

Step 4: The min-max problem in Eq. (6) is transformed into the equivalent single-objective problem by introducing a

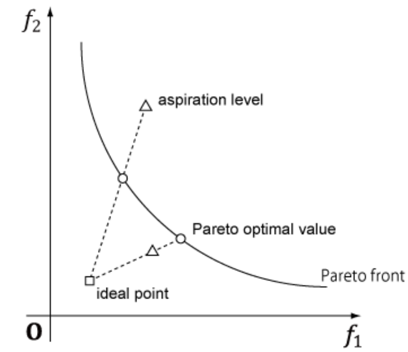


Fig. 4. Pareto optimal solution when searching by STOM

new design variable y as follows.

$$\begin{aligned}&\text{Minimize:} \quad y \\ &\text{subject to:} \quad w_i\left(f_i(\boldsymbol{x},\boldsymbol{z})-f_i^I\right)\le y, \quad (j=1,\cdots,k) \\ &\qquad g_j(\boldsymbol{d},\boldsymbol{z})<0, \quad (j=1,\cdots,m) \\ &\qquad x_i^L\le x_i\le x_i^U \quad (i=1,\cdots,n_x)\end{aligned} \tag{7}$$

Step 5: If the objective function values are satisfactory, the search is finished. Otherwise, update the aspiration level f_i^A and return to Step 2.

The weight coefficient w_i plays an important role in obtaining the Pareto solution in the direction of the aspiration level, which is directly related to the designer's preference. As shown in Fig. 4, the Pareto optimal solution is usually located on the line connecting the ideal point and the aspiration level in the objective function space, regardless of whether or not the aspiration level lies in the feasible region. When Eq. (7) is solved using a nonlinear programming method, an accurate Pareto optimal solution is obtained.

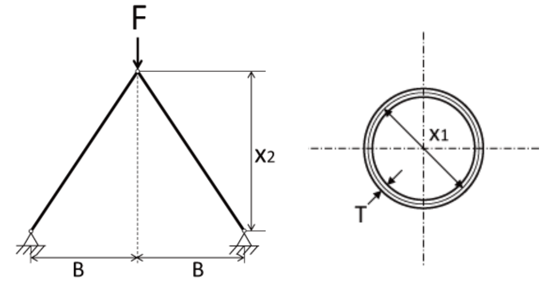
An accurate Pareto set is obtained by parametrically changing the aspiration level. On the other hand, the designers can investigate only the desired region in detail by arranging the aspiration level properly without obtaining the full Pareto set.

4. Numerical Examples

4.1 Example 1: two-bar truss structure

Consider the following problem involving the two-bar truss structure shown in Fig. 5 [9]. The original design problem is to find the nominal diameter for member x_1 and the height of structure x_2 to minimize the element stress under the constraints of the total volume $g_1(\boldsymbol{x})$ and buckling stress $g_2(\boldsymbol{x})$. The external force F is set at 150 kN, the thickness of the circular tube T is set at 2.5 mm, the width of structure B is set at 750 mm, and the elastic modulus E is set at 210 GPa. The original optimization problem is formulated as follows:

$$\text{Minimize:} \quad f(\boldsymbol{x})=\frac{F\sqrt{B^2+x_2^2}}{2\pi T x_1 x_2} \tag{8}$$



(a) Truss structure (b) Cross-sectional shape

Fig. 5. Two-bar truss design problem

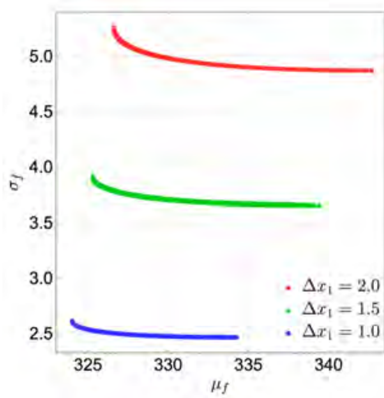
$$\begin{aligned}&\text{subject to:} \quad g_1(\boldsymbol{x})=\frac{2\pi T x_1\sqrt{B^2+x_2^2}}{70000}\le 1 \\ &\qquad g_2(\boldsymbol{x})=\frac{4F_1\left(B^2+x_2^2\right)^{3/2}}{E\pi^3 T x_1 x_2\left(T^2+x_1^2\right)}\le 1 \\ &\qquad 1\le x_1\le 100 \\ &\qquad 1\le x_2\le 1000\end{aligned}$$

The robust design problem is considered when the member diameter and height have uncertainties, with (Δx_1, Δx_2) used to denote random variables, where the mean value of x_1 and x_2 are treated as design variables. In this example, the standard deviation of a random variable is assumed to be $\Delta x_i/3$ [9]. The robust multiobjective design problem is formulated as follows:

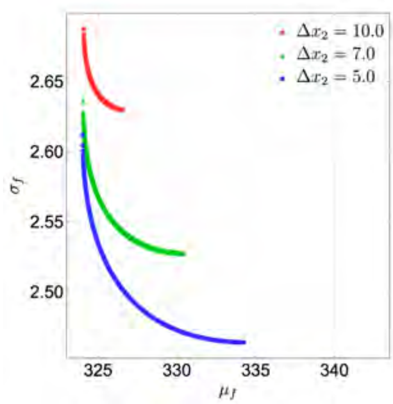
$$\begin{aligned}&\text{Minimize:} \quad f_1(\boldsymbol{x})=f(E[\boldsymbol{x}])=\frac{F\sqrt{B^2+x_2^2}}{2\pi T x_1 x_2} \\ &\qquad f_2(\boldsymbol{x},\Delta\boldsymbol{x})=\sqrt{\mathrm{Var}[f(\boldsymbol{x},\Delta\boldsymbol{x})]} \\ &\qquad =\frac{1}{3}\sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2\Delta x_1^2+\left(\frac{\partial f}{\partial x_2}\right)^2\Delta x_2^2} \\ &\text{subject to:} \quad \hat{g}_1(\boldsymbol{x})=g_1(E[\boldsymbol{x}])+\sqrt{\mathrm{Var}[g_1(\boldsymbol{x})]}\le 1 \\ &\qquad \hat{g}_2(\boldsymbol{x})=g_2(E[\boldsymbol{x}])+\sqrt{\mathrm{Var}[g_2(\boldsymbol{x})]}\le 1 \\ &\qquad 1+\Delta x_1\le x_1\le 10-\Delta x_1 \\ &\qquad 10+\Delta x_2\le x_2\le 100-\Delta x_2\end{aligned} \tag{9}$$

where $\hat{g}_j(\boldsymbol{x})$ are the constraints for the considered variations of the design variables evaluated as the first-order approximation [8]. The constraints mean the volume and buckling load limits should be satisfied under variations.

The Pareto front under several values of $\Delta\boldsymbol{x}$ is obtained by parametrically changing the aspiration level, as shown in Fig. 6. The ideal points are set by solving each single-objective function problem, as listed in Table 1. Fig. 6 (a) shows the Pareto set when changing Δx_1 to 1.0, 1.5, and 2.0, where Δx_2 is constant at Δx_2 = 5.0. On the other hand, Fig. 6 (b) corresponds to the Pareto set when changing Δx_2 to 5.0, 7.0, and 10.0, where Δx_1 is constant at Δx_1 = 1.0. Note that the blue



(a) Changing Δx_1 under $\Delta x_2 = 5.0$



(b) Changing Δx_2 under $\Delta x_1 = 1.0$

Fig. 6. Pareto front in example 1

curves in both figures indicate the same Pareto set under (Δx_1, Δx_2) = (1.0, 5.0). It is found that smooth Pareto fronts are obtained in both cases, because each Pareto design is obtained by using a sequential quadratic programming method.

As Δx_1 becomes larger, the Pareto front moves in the upper right direction, as shown in Fig. 6 (a). On the other hand, the Pareto front moves in the upper left direction as Δx_2 becomes larger, as shown in Fig. 6 (b).

A comparison of these figures shows that the variation of x_1 has a larger effect on the Pareto set, especially on f_2, which is the variation of the normal stresses.

4.2 Example 2: ten-bar truss structure

Consider the ten-bar truss problem shown in Fig. 7, where the representative length L is set at 360, and the applied load P is 100 [10]. The original deterministic design problem is formulated to minimize the structural volume in terms of the member's cross-sectional area subject to the member's stress constraints.

This problem is modified to give the following robust multiobjective optimization problem. Assume that the applied load P and member strength have variations and are treated as random variables. The robust design problem is formulated as three objective function problems, which are defined as the structural volume and the mean and standard deviation of the

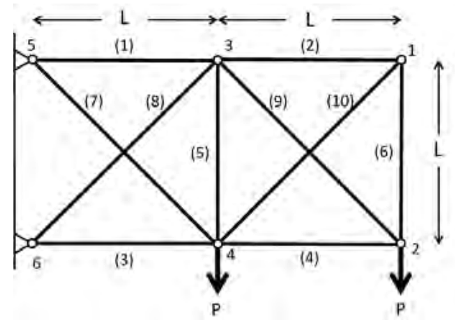


Fig. 7. Ten-bar truss design problem

Table 1. Ideal points for each condition in example 1.

Δx_1	Δx_2	f_1^I	f_2^I
1.0	5.0	324.02	2.46
1.5	5.0	325.31	3.65
2.0	5.0	326.61	4.87
1.0	7.0	324.05	2.53
1.0	10.0	324.09	2.62

tip displacement d_2 in terms of the member's cross-sectional area x_i, where x_i is assumed to be deterministic. The robust multiobjective optimization problem is formulated as follows:

$$\begin{aligned}
\text{Minimize:}\quad & f_1(\boldsymbol{x}) = \sum_{i=1}^{10} l_i x_i && (10)\\
& f_2(\boldsymbol{x},\boldsymbol{z}) = E[d_2(\boldsymbol{x},\boldsymbol{z})] \\
& f_3(\boldsymbol{x},\boldsymbol{z}) = \sqrt{\mathrm{Var}[d_2(\boldsymbol{x},\boldsymbol{z})]} \\
\text{subject to:}\quad & -s + \Delta s \le \sigma_i(\boldsymbol{x},\boldsymbol{z}) \le s - \Delta s \quad (i = 1,\cdots,10) \\
& 0.1 \le x_i \le 10.0 \qquad (i = 1,\cdots,10)
\end{aligned}$$

where l_i is the member length, σ_i is the member stress, and s and Δs are the mean value and standard deviation of the member strength, respectively. The mean value and standard deviation of the applied load are set at $E[P]$ = 100.0 and ΔP = 10.0, respectively. The mean value and standard deviation of the member strength are set at $E[s]$ = 25.0 and Δs = 1.0, respectively.

The obtained Pareto set in the objective function space is shown in Fig. 8 with different viewing angles, where each point is obtained by changing the aspiration level parametrically under the ideal point $\boldsymbol{f}^I = (1.659 \times 10^4, 1.956, 1.345 \times 10^{-2})^T$, as determined from each single-objective optimization problem. It is found that a smooth Pareto surface is obtained, because each Pareto design is obtained by using a sequential quadratic programming method. The Pareto set indicates that a trade-off exists between the structural volume, mean of the tip displacement, and variance of the tip displacement.

Next, the correspondences of the selected Pareto designs are investigated in detail. First, designs A, B, C, and D on the Pareto set shown in Fig. 8 are selected, because the Pareto solutions have almost the same volume and mean of the tip displacement, but have different standard deviations of the tip displacement, as shown in Fig. 9. Design A is the most robust

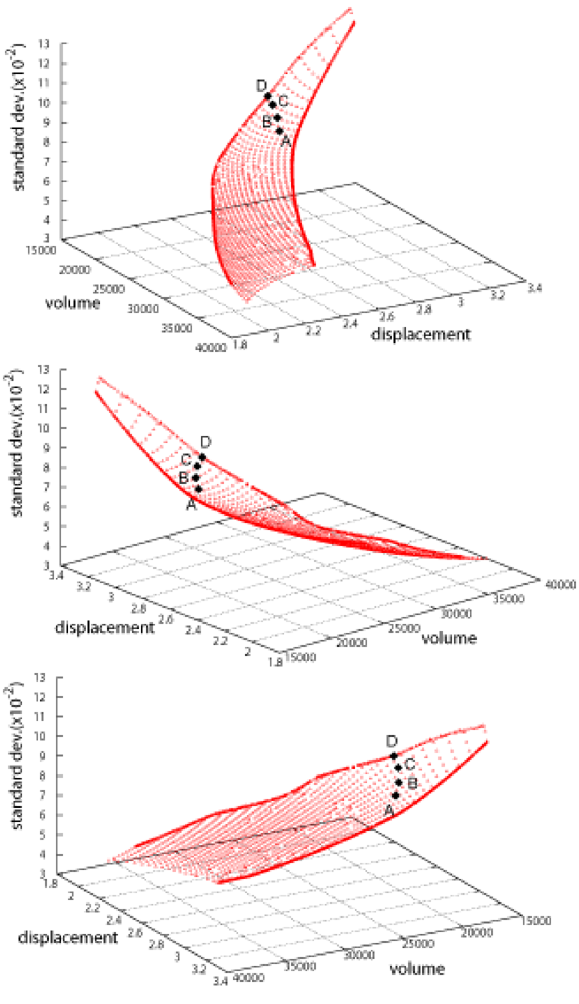


Fig. 8. Pareto front in example 2

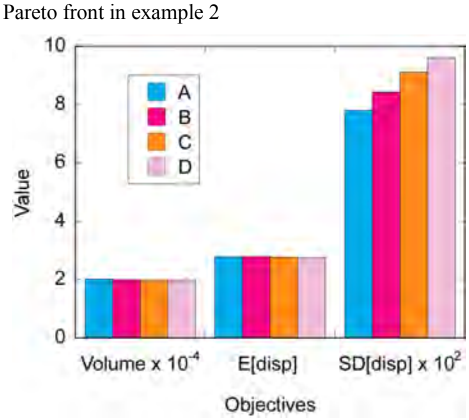


Fig. 9. Comparison of Pareto designs

among these four designs, because the variation of the tip displacement is the smallest. The aspiration levels (Asp.) and obtained objective function values (Pareto) are listed in Table 2. This investigation is possible without obtaining full Pareto set actually, if the aspiration levels are adequately selected for obtaining these four designs.

Fig. 10 shows the changes in the member cross-sectional areas of these designs, where the horizontal axis indicates the standard deviation of the tip displacement $f_3(\boldsymbol{x})$. The leftmost

Table 2. Objective function values of selected Pareto designs

Design		Volume $\times 10^4$	$E[d_2]$	$\sqrt{\mathrm{Var}[d_2]}$ $\times 10^{-2}$
A	Asp.	1.250	1.400	1.943
	Pareto	2.023	2.792	**7.808**
B	Asp.	1.202	1.426	2.166
	Pareto	1.995	2.790	**8.430**
C	Asp.	1.210	1.450	2.450
	Pareto	1.982	2.770	**9.108**
D	Asp.	1.200	1.400	2.600
	Pareto	1.992	2.741	**9.622**

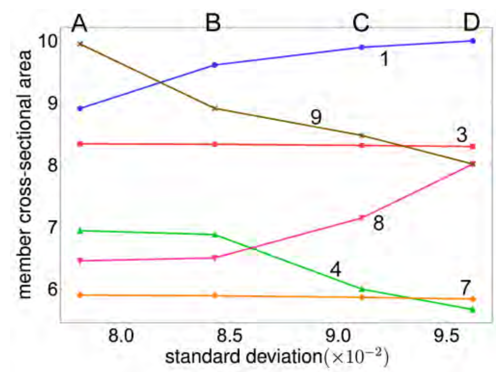


Fig. 10. Transitions of member cross-sectional areas for designs A, B, C, and D

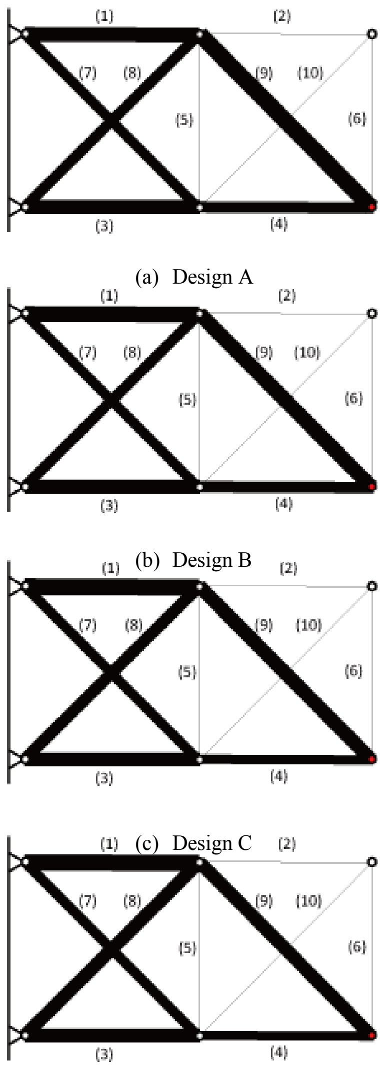
Table 3. Comparison of cross-sectional areas

	A	B	C	D	change
x_1	8.911	9.611	9.897	10.0	↗
x_2	0.1	0.1	0.1	0.1	
x_3	8.341	8.334	8.316	8.299	→
x_4	6.942	6.879	6.000	5.672	↘
x_5	0.1	0.1	0.1	0.1	
x_6	0.1	0.1	0.1	0.1	
x_7	5.904	5.894	5.868	5.841	→
x_8	6.456	6.503	7.147	8.016	↗
x_9	9.951	8.914	8.476	8.017	↘
x_{10}	0.1	0.1	0.1	0.1	

value corresponds to design A, the most robust design, and the rightmost value corresponds to design D. The cross-sectional areas of members 2, 5, 6, and 10 are not shown in this figure, because these areas converge to the lower bound (0.1) in all the designs. The details of the cross-sectional area values are listed in Table 3. As the variation of the tip displacement becomes larger, the cross-sectional areas of members 1 and 8 increase, and those of 4 and 9 decrease. The cross-sectional area distributions of designs A, B, C, and D are illustrated in Fig. 11. Note that members 1 and 8 are located to the left fixed side, but 4 and 9 are to the right free side. In other words, the variation of the tip displacement decreases when the cross-sectional area of the tip-side member increases.

5. Conclusions

This paper proposed a robust multiobjective design method that uses the STOM [4]. Through numerical examples, the



(a) Design A

(b) Design B

(c) Design C

(d) Design D

Fig. 11. Cross-sectional area distributions

usefulness of this method was demonstrated.

- An accurate Pareto set is obtained by parametrically changing the aspiration level, because each Pareto solution is obtained using a mathematical programming method.
- It was shown that the proposed method could be used to investigate the effect of the variation of random variables on the shape of the Pareto frontier. In addition, the shift of each Pareto solution with the same aspiration level could be traced with respect to the variation of uncertain parameters.
- This method makes it possible to investigate how the variation of the design variables and parameters affect the Pareto set. This investigation is possible without obtaining full Pareto set.

Acknowledgment

Part of this research was supported by JSPS KAKENHI 26249131. The authors wish to express their appreciation to Prof. S. Kitayama at Kanazawa University for his valuable comments on this research.

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