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著者	Ito Makoto, Kogiso Nozomu
journal or publication title	Journal of Advanced Mechanical Design, Systems, and Manufacturing
volume	10
number	6
page range	JAMDSM0083
year	2016-10-13
権利	(c) 2016 The Japan Society of Mechanical Engineers
URL	http://hdl.handle.net/10466/15695

doi: 10.1299/jamdsm.2016jamdsm0083

Information Uncertainty Evaluated by Parameter Estimation and Its Effect on Reliability-based Multiobjective Optimization

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Received XX May 2016

Abstract

Reliability-based multiobjective optimization (RBMO) is a method that integrates multiobjective optimization with a reliability analysis. The method is useful for a large or complicated design problem such as aerospace structure design. Reliability analysis generally requires the probabilistic distribution parameters of random variables such as the mean and standard deviation. However, for an actual design problem, the probabilistic parameters are sometimes estimated with insufficient accuracy because of a limited number of experiments. In that case, the uncertainty in the distribution parameter is not negligible. This study proposes the evaluation method to estimate the effect of the information uncertainty at first, where the uncertainty is evaluated by using the confidence interval. Some numerical examples illustrate the effectiveness of the proposed method in comparison with a conventional method, Gibbs sampling. Then, the effect of the parameter uncertainty on the RBMO is illustrated through numerical examples. The RBMO problem is formulated by using the satisficing trade-off method (STOM), where the multiobjective optimization problem is transformed into the equivalent single-objective optimization method. For the reliability-based design optimization, a modified SLSV (single-loop-single-vector) method is adopted for the computational efficiency. The effects of the parameter uncertainty on the selected Pareto solutions according to the aspiration level are investigated by using the confidence intervals of the Pareto solutions.

Key words : Parameter Estimation, Information Uncertainty, Bayesian Statistics, Reliability-based Multiobjective Optimization, Satisficing Trade-off Method

1. Introduction

The reliability of huge and complicated structural designs, such as aerospace structures, has a significant effect on design problems that require decision-making under uncertainties. Many design optimization methodologies with reliability theory have been developed as reliability-based design optimization (RBDO) [Aouse and Chateauf, 2010]. RBDO generally requires the probabilistic parameters of random variables to evaluate the reliability. For example, the first-order reliability method (FORM) [Madsen et al., 1986] converts the probabilistic distribution of random variables into a standardized normal distribution to evaluate the probability of failure.

Recently, reliability-based multiobjective optimization (RBMO), which integrates the multiobjective optimization method with reliability analysis, was developed. Many researchers have focused on evolutionary algorithms such as multiobjective genetic algorithms [Sinha, 2007] and multiobjective particle swarm optimization (MOPSO) [Reyes-Sierra and Coello Coello, 2006]. One of the authors previously proposed a hybrid-type MOPSO for RBMO problems [Kogiso et al., 2012a, Kogiso et al., 2013] to improve the computational efficiency. However, the computational efficiency of RBMO remains inferior to that of mathematical programming methods. Then, one of the authors proposed a new method by applying the satisficing trade-off method (STOM) to the RBMO problem [Kogiso et al., 2014].

However, in actual design problems, the probabilistic parameters of random variables are sometimes estimated with insufficient accuracy, because the number of experiments is limited. In this case, the probabilistic parameters such as the mean and standard deviation also have uncertainties. This uncertainty of information is not negligible for design

problems with high accuracy requirements. However, it may be prohibitive to increase the number of experiments in actual situations.

Instead, the uncertainty should be reduced by adopting a data assimilation method. This type of statistics modeling is also employed for estimating the distribution of failure probability. [Gunawan and Papalambros, 2006] proposed a method for considering the uncertainties surrounding incomplete information and assumed the reliability to be estimated by a beta distribution. [Wang et al., 2009] obtained an extreme case of the beta distribution from insufficient information by applying Bayes' theorem and extreme distribution theory. [Cho et al., 2012, Cho et al., 2014] proposed a method to estimate the confidence level of the reliability. They showed that Bayesian statistics is a useful tool for estimating the probabilistic parameter and that the Bayesian updating method improves the estimated reliability with sufficient accuracy. However, for methods using Bayes' theorem, it is possible to improve the estimation accuracy of the reliability from a limited number of sample data.

The authors previously proposed a new method to estimate the confidence interval of the reliability [Ito and Kogiso, 2015]. In this paper, this method is applied to the RBMO using STOM considering the information uncertainty. Through numerical examples, the effect of information uncertainties of the distribution parameters on the Pareto solutions obtained by the RBMO approach is investigated through the confidence interval of the Pareto solutions.

This paper is organized as follows. The RBMO using STOM as the multiobjective optimization method is reviewed in the next section. Then, the proposed method is described in Section 4 and the validity of the proposed method is illustrated through simple numerical examples in Section 5. Finally, several conclusions are remarked.

2. Nomenclature

d	Design variable vector
D	Experimental data vector [$M \times 1$] generated by x
\bar{D}	Sample mean of D
\tilde{D}	Variance of D
F	Objective function vector
f_i	The i -th objective function
f_i^*	The ideal points of the i -th objective function
\tilde{f}_i^A	The aspiration level of the i -th objective function
g_j	The j -th constrained function that corresponds to the failure mode in the reliability analysis
k_{\max}	Maximum iteration number of Gibbs sampling
k_b	Number of rejected samples
m	Number of failure modes
M_D	Number of chosen \tilde{D}
n	Number of the design variables
N_D	Number of data
N_{set}	Number of "data set"
r	Number of objective functions
x	Random variable vector
\tilde{x}	Deviation of x
w_i	The weighting factor of of the i -th objective function
α	Hyper parameter (shape parameter) of σ^2
β	Hyper parameter (scale parameter) of σ^2
β_j^T	Target reliability of the j -th failure mode
μ	Probabilistic parameter (mean value) of x
μ_0^*	Mean value of each "data set"
ν_0	Hyper parameter (mean value) of μ
σ^2	Probabilistic parameter (mean value) of x
σ_0^{2*}	Variance of each "data set"
τ_0	Hyper parameter (variance) of μ

3. Satisficing Trade-off Method (STOM)

3.1. Formulation of STOM in Multiobjective Optimization

The multiobjective optimization problem is intended to simultaneously optimize multiple objective functions described as follows:

$$\mathbf{F}(\mathbf{d}) = [f_1(\mathbf{d}), f_2(\mathbf{d}), \dots, f_r(\mathbf{d})]^T \quad (1)$$

where r is the number of objective functions, $\mathbf{d} = [d_1, d_2, \dots, d_n]$ are the design variables and n is the number of design variables.

Then, the multiobjective optimization problem is formulated as follows:

$$\begin{aligned} \text{Minimize : } & \mathbf{F}(\mathbf{d}) = [f_1(\mathbf{d}), f_2(\mathbf{d}), \dots, f_r(\mathbf{d})]^T \\ \text{subject to : } & g_j(\mathbf{d}) \geq 0 \quad (j = 1, 2, \dots, m) \\ & \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \end{aligned} \quad (2)$$

where $g_j(\mathbf{d})$ is the j -th constrained function that corresponds to the j -th failure mode in the reliability analysis. \mathbf{d}^L and \mathbf{d}^U are the upper and lower bounds of the design variables, respectively. In this study, STOM [Kogiso et al., 2014] is applied to solve the multiobjective optimization problem. The algorithm of STOM is described as follows.

Step 1 The ideal points $f_i^* (i = 1, 2, \dots, r)$ of the objective functions are determined. Generally, the ideal point f_i^* is set as the optimal point of the single objective problem optimization corresponding to each objective function. Then, the aspiration level \tilde{f}_i^A , ($i = 1, 2, \dots, r$) is set as each objective function.

Step 2 The following weighting factor is introduced for each objective function:

$$w_i = \frac{1}{\tilde{f}_i^A - f_i^*} \quad (i = 1, 2, \dots, r) \quad (3)$$

Step 3 The multiobjective optimization problem is set as the following Chebyshev norm problem.

$$\begin{aligned} \text{Minimize : } & \max_{1 \leq i \leq r} w_i f_i(\mathbf{d}) \\ \text{subject to : } & g_j(\mathbf{d}) \geq 0 \quad (j = 1, 2, \dots, m) \\ & \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \end{aligned} \quad (4)$$

Step 4 Equation (4) is transformed into an auxiliary single optimization problem and is solved by a conventional single objective optimization algorithm. In this study, sequential quadratic programming (SQP) is adopted. The formulation of the auxiliary optimization problem is described as follows:

$$\begin{aligned} \text{Minimize : } & z + y \sum_{i=1}^r w_i f_i(\mathbf{d}) \\ \text{subject to : } & w_i (f_i(\mathbf{d}) - \tilde{f}_i^*) \leq z \quad (i = 1, 2, \dots, r) \\ & g_j(\mathbf{d}) \geq 0 \quad (j = 1, 2, \dots, m) \\ & \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U, \end{aligned} \quad (5)$$

where y is a small positive number used to avoid convergence to weak Pareto solutions.

Step 5 If the user is satisfied with the optimum solution obtained from the equivalent problem, terminate the search process; otherwise, update the aspiration level $\tilde{f}_i^A (i = 1, 2, \dots, r)$ and return to Step 2.

3.2. RBMO using STOM

RBMO is formulated as follows:

$$\begin{aligned} \text{Minimize : } & \mathbf{F}(\mathbf{d}) = [f_1(\mathbf{d}), f_2(\mathbf{d}), \dots, f_r(\mathbf{d})] \\ \text{subject to : } & P[g_j(\mathbf{x}) \leq 0] \leq \Phi(-\beta_j^T) \quad (j = 1, \dots, m) \end{aligned} \quad (6)$$

$$\mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U,$$

where β_j is the j -th target reliability index and \mathbf{x} and \mathbf{d} are random variables and design variables, respectively. In this paper, the values of \mathbf{x} are assumed to be independent from each other and to follow a normal distribution. In many cases, the mean values of the random variables are adopted as the design variables.

In Eq. (6), the probability of failure is given by the following integral:

$$P[g_j(\mathbf{d}, \mathbf{x}) \leq 0] = \int \cdots \int_{g_j(\mathbf{d}, \mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}, \tag{7}$$

where $f_{\mathbf{X}}(\mathbf{X})$ is the joint density function. The difficulty of calculating this integral has led to the use of approximate methods such as the first-order reliability method (FORM [Madsen et al., 1986]).

The solution of problem (7) is generally given by an approximation approach such as the double-loop approach (e.g. RIA (reliability index approach) [Enevoldsen and Sorensen, 1994], PMA (performance measure approach) [Tu et al., 1999]). As the double-loop approach requires computational effort, the following efficient approaches such as mono-level approach [Kuschel and Rackwitz, 1997], the single loop approach (e.g. SLSV (single-loop-single-vector) [Chen et al., 1997]) and decoupled approach (e.g. SORA (sequential optimization and reliability analysis) [Du and Chen, 2004], SAP (sequential approximate programming strategy) [Chen et al., 2006]) have been developed.

This study adopts the single-level approach. This procedure transforms the RBDO problem into a deterministic optimization problem and converts the double-loop optimization loop into the single-loop optimization. In this study, the modified-SLSV [Kogiso et al., 2012b] is adopted as the single-loop procedure of the RBDO algorithm. The original SLSV method converts the double-loop optimization loop of the RBDO into a single-loop approach by approximating the most probable point (MPP). The modified-SLSV method improves the convergence by eliminating zigzagging iteration to avoid divergence in the search for the optimum design. The formulation of STOM with the modified-SLSV method is formulated as follows. Further details have been published in references [Chen et al., 1997, Kogiso et al., 2012b].

$$\begin{aligned} \text{Minimize : } & z + y \sum_{i=1}^r w_i f_i(\mathbf{x}) & (8) \\ \text{subject to : } & w_i(f_i(\mathbf{d}) - \bar{f}_i^*) \leq z \quad (i = 1, 2, \dots, r) \\ & g_j(\mathbf{d}, \mathbf{d} - \beta_j^T \boldsymbol{\sigma} \boldsymbol{\alpha}_j^*) \leq 0 \quad (j = 1, \dots, m) \\ & \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \\ \text{where : } & \boldsymbol{\alpha}_j^* = \frac{\boldsymbol{\sigma}_j \nabla g_j(\boldsymbol{\mu}, \mathbf{x}_j^*)}{|\boldsymbol{\sigma} \nabla g_j(\boldsymbol{\mu}, \mathbf{x}_j^*)|} \quad (j = 1, \dots, m) \end{aligned}$$

In Eq. (8), $\boldsymbol{\mu}$ is the mean value of the random vector \mathbf{x} that is allocated as design variables \mathbf{d} . $\boldsymbol{\sigma}$ is a diagonal matrix whose diagonal element consists of the standard deviation of each random variable.

4. Parameter Estimation Method

4.1. Estimation of Hyper Parameter

Assume that we have N_D experimental data with variations for random variables and the experimental data set \mathbf{D} follows a normal distribution. When N_D is not large, the differences between the sample parameter (mean and variance) and the population parameter may not be negligible. Focusing on the uncertainty of the variance, the modified experimental data $\tilde{\mathbf{D}}$ are introduced by shifting the sample mean $\bar{\mathbf{D}}$.

$$\tilde{\mathbf{D}} = \mathbf{D} - \bar{\mathbf{D}} \tag{9}$$

Sampling is used to estimate the unknown parameter by choosing M_D data from $\tilde{\mathbf{D}}$ without repetition. This combination is known as the “data set” and the number of “data sets” is denoted as $N_{\text{set}} (= N_D C_{M_D})$. Each “data set” has its own estimated mean value and variance, $(\mu_1^*, \dots, \mu_{N_{\text{set}}}^*)$ and $(\sigma_1^{2*}, \dots, \sigma_{N_{\text{set}}}^{2*})$. From the N_{set} mean values and variances, the prior estimation of the hyper parameters are estimated by using the maximum likelihood method. This estimation process is illustrated in Fig. 1.

Under the assumption that the mean value follows a normal distribution, the hyper parameters are estimated as follows:

$$v_0 = \frac{1}{N_{\text{set}}} \sum_{i=1}^{N_{\text{set}}} \mu_i^* \tag{10}$$

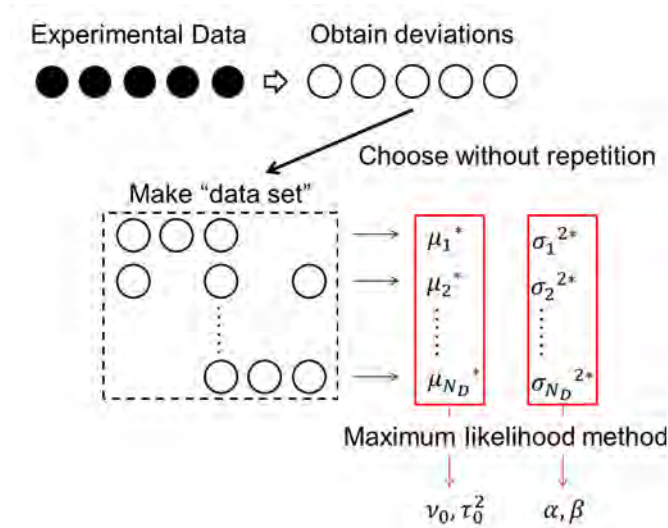


Fig. 1 Estimating the hyper parameter from limited data

$$\tau_0^2 = \frac{1}{N_{\text{set}}} \sum_{i=1}^{N_{\text{set}}} (\mu_i^* - v_0)^2 \tag{11}$$

Under the assumption that the variance follows an inverse gamma distribution, it is too difficult to use the maximum likelihood method directly. Therefore, the hyper parameters are estimated by the solution of the maximum optimization problem as follows:

$$\begin{aligned} \text{Maximize : } & \prod_{i=1}^{N_{\text{set}}} f(\alpha, \beta, \sigma_i^{2*}) \\ \text{where : } & f(\alpha, \beta, \sigma_i^{2*}) = \frac{\beta^\alpha}{\Gamma(\alpha)} (\sigma_i^{2*})^{-(\alpha+1)} \exp(-\beta/\sigma_i^{2*}) \\ \text{subject to : } & \alpha > 0 \quad \beta > 0 \end{aligned} \tag{12}$$

Sequential quadratic programming (SQP) is also adopted to obtain the hyper parameters in this study.

Then, the mode of the variance σ^2 is estimated using α and β as follows:

$$\sigma_{\text{mode}}^2 = \frac{\beta}{\alpha + 1} \tag{13}$$

4.2. Sample Sequence Creation

The probabilistic parameters are estimated by creating sample sequences using Gibbs sampling. As shown above, the hyper parameter of the Gibbs sampling has already been obtained. However, we assumed that the number of experimental data points are insufficient to obtain the estimated value properly.

It is important to confirm the difference between the estimated parameter and initial data. In this study, the log likelihood is evaluated for each sample. Under the assumption that the variance of experimental data \tilde{D} follows a normal distribution, the log likelihood is formulated as follows:

$$L(\mu^{(k)}, \sigma^{2(k)}, \tilde{D}) = -\frac{N_D}{2} \log(2\pi\sigma^{2(k)}) - \frac{1}{2\sigma^{2(k)}} \sum_{i=1}^{N_D} (\tilde{D}_i - \mu^{(k)})^2, \tag{14}$$

where k is the sample number. If the log likelihood becomes worse than the previous value, the sample is rejected by a probability of 90%.

The above process is summarized as follows.

- (1) Set the initial value of $\mu^{(k)}$ and $\sigma^{2(k)}$, where k is set to 1. In this study, these initial values are set as $\mu^{(k)} = v_0$ and $\sigma^{2(k)} = \beta/(\alpha + 1)$.
- (2) $\mu^{(k+1)}$ is generated by a series of random numbers, which follows the conditional posterior distribution, $f(\mu^{(k+1)} | \sigma^{2(k)}, \tilde{D})$.

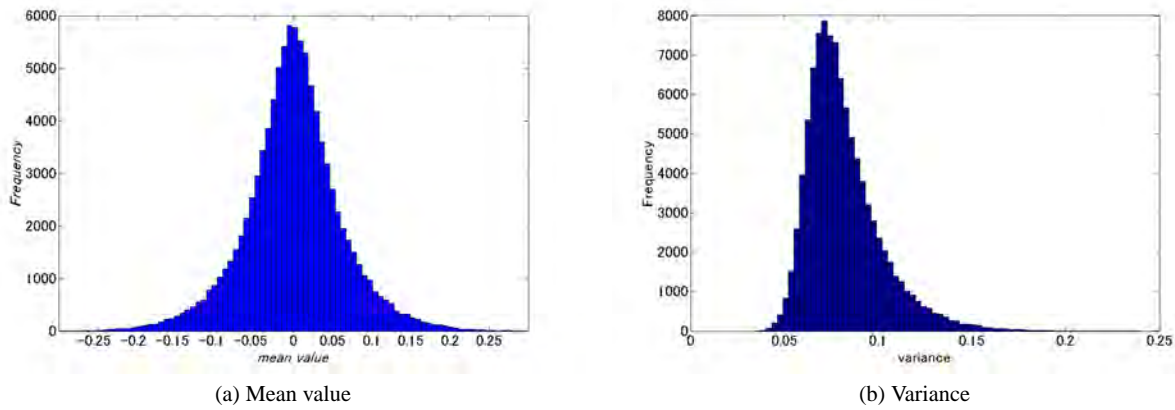


Fig. 2 Histogram of mean value and variance by the proposed method

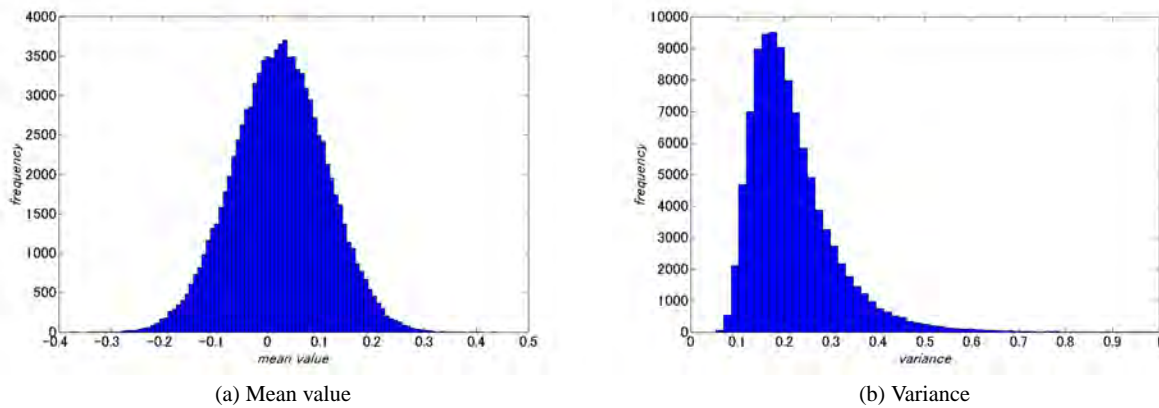


Fig. 3 Histogram of mean value and variance by Gibbs sampling

Table 1 Comparison of the 90 % confidence interval of probabilistic parameter estimated using the proposed method

	Mean value	Variance
Proposed method	(-0.1037, 0.1023)	(0.05726 0.1205)
Gibbs sampling	(-0.1240, 0.1690)	(0.1116, 0.3889)

(3) $\sigma^{2(k+1)}$ is generated by a series of random numbers, which follows the conditional posterior distribution, $f(\sigma^{2(k+1)}|\mu^{(k+1)}, \tilde{D})$.

(4) The log likelihood is evaluated. If the log likelihood becomes worse than the previous value, the sample is rejected by a probability of 90%.

(5) Until the iteration number k reaches the maximum value k_{max} , repeat the above step. Otherwise, the initial sample sequence $k = 1$ to k_b is rejected. Then, the confidence interval of μ and σ^2 is evaluated from the remaining sample sequence.

For example, it is considered that the probabilistic parameters are estimated using the following experimental data of X , which follow a normal distribution:

$$D = (-0.2842 \quad 0.1620 \quad -0.0650 \quad 0.3567 \quad 0.3951)^T \tag{15}$$

Actually, these data are obtained from the normally distributed random number set, $N(0, 0.3^2)$. However, the estimation is performed under the assumption that we don't know the fact.

The distribution of the sample mean and variance and the confidence interval obtained by the proposed method are shown in Fig. 2, where $M_D=3$, $k_{max}=100000$, and $k_b=1000$. Figure 2 (a) and (b) show the histogram of μ and σ^2 , respectively. For comparison, histograms of μ and σ^2 evaluated by a general Gibbs sampling method is illustrated in Figure 3 (a) and (b).

The 90% confidence intervals of the mean value and variance determined by the proposed method is compared with those by the Gibbs sampling in Table 1. The 90% confidence interval by the proposed method includes the true value of

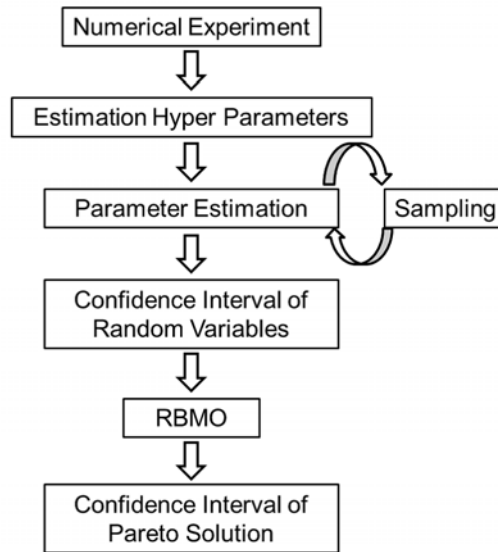


Fig. 4 Flow of the proposed method

0.3². On the other hand, Gibbs sampling does not include the true value. In addition, the mean value range indicates that the proposed method is more accurate than Gibbs sampling.

Finally, the flow of the proposed method including RBMO is illustrated in Fig. 4. After evaluating the confidence interval of the random variables, the variance of the optimum design considering the uncertainty in the information is investigated. This is evaluated as the confidence interval of the Pareto solution of the RBMO problem.

5. Numerical Examples

The effect of the parameter uncertainties on the RBMO is investigated through numerical examples, where the parameters are set as $N_D=5$, $M_D=3$, $k_{max}=100000$, and $k_b=1000$.

5.1. Mathematical problem

The following two-dimensional mathematical RBMO problem is considered:

$$\begin{aligned}
 &\text{Minimize : } f_1(\mathbf{d}) = 3d_1 + d_2 && (16) \\
 &f_2(\mathbf{d}) = -d_1 + d_2 + 10 \\
 &\text{subject to : } P[g_j(\mathbf{x}) \leq 0] \leq \Phi(-\beta_j^T) \quad (j = 1, \dots, 3) \\
 &g_1(\mathbf{x}) = \frac{x_1^2 x_2}{20} - 1 \\
 &g_2(\mathbf{x}) = \frac{(x_1 + x_2 - 5)^2}{30} + \frac{(x_1 - x_2 - 12)^2}{120} - 1 \\
 &g_3(\mathbf{x}) = \frac{80}{x_1^2 + 8x_2 + 5} - 1 \\
 &0 \leq d_i \leq 10 \quad (i = 1, 2),
 \end{aligned}$$

where \mathbf{x} are the normally distributed random variables; the design variables \mathbf{d} are set as the mean value of the random variables, $\mathbf{d} = \boldsymbol{\mu} = (\mu_1, \mu_2)^T$; and the target reliability is set to $\beta_j^T = 3.0$. This problem is an extension of the RBDO problem in [Liang et al., 2004]. The standard deviation of x_1 is assumed to be unknown, but that of x_2 is known as $\sigma_2 = 0.3$. Here, the standard deviation of x_1 should be estimated from a limited number of the following experimental data in Eq. (15), where the five values are obtained from the normally distributed random number $N(0.0, 0.3^2)$ because the original problem uses $\sigma_1 = 0.3$. The distribution parameters of x_1 are first evaluated by the proposed method. Table 2 lists the sample values, the true value, and the confidence interval of the estimated parameters. It is found that the true value is included in the 90% confidence interval.

Table 2 Estimated parameter values in mathematical problem

	Sample value	True value	CI* (90%)
μ	0.1129	0.0	(-0.1037, 0.1023)
σ	0.2876	0.3	(0.2393, 0.3471)

CI* means "confidence interval".

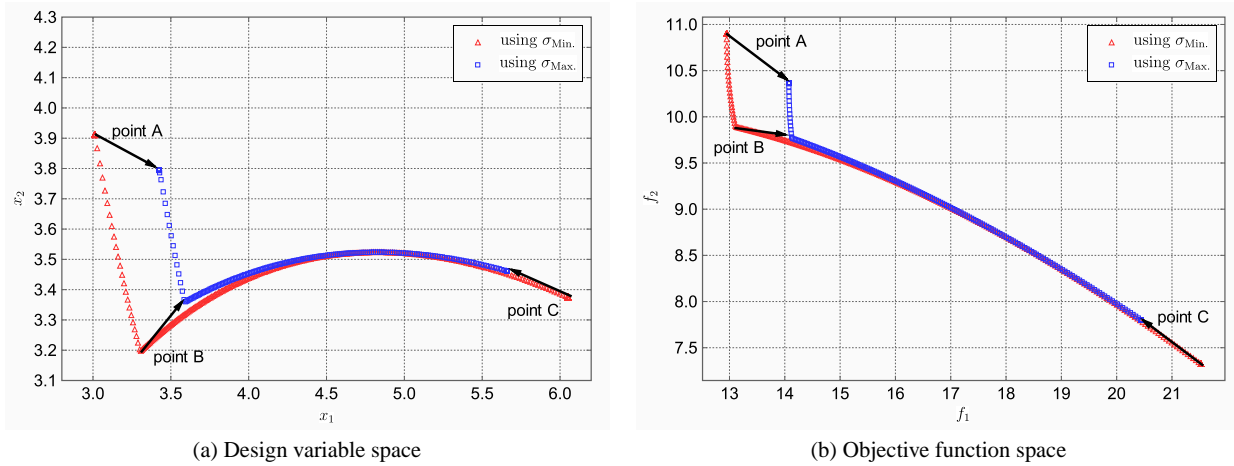


Fig. 5 Confidence interval of Pareto solution in mathematical problem

Table 3 Change on typical three Pareto solutions from minimum to maximum value of σ_1 in confidence interval range in design variable space and objective function space

	Variable space			Function space		
	Δd_1	Δd_2	$\Delta d_2/\Delta d_1$	Δf_1	Δf_2	$\Delta f_2/\Delta f_1$
Point A	0.411	-0.117	0.285	1.096	-0.513	-0.468
Point B	0.283	0.173	0.611	1.006	-0.108	-0.107
Point C	-0.416	0.093	-0.285	-1.066	0.459	-0.431

Then, the Pareto optimum solutions are obtained using the estimated parameters. The Pareto solutions corresponding to the confidence interval of σ_1 are shown in Fig. 5. In these figures, σ_{Min} and σ_{Max} correspond to the Pareto set obtained for minimum and maximum values of σ_1 in the confidence interval range, respectively. In Fig. 5, change of the typical three Pareto solutions the leftmost point (Point A), the kink point (Point B) and the rightmost point (Point C) are indicated by arrows. Then, the changes of the design variables and the objective function values are listed in Table 3. As indicated in this table, all absolute values of $\Delta d_2/\Delta d_1$ and $\Delta f_2/\Delta f_1$ in three points are less than 1. It means that the effects of change of σ_1 on d_1 and f_1 are larger than those of d_2 and f_2 , respectively. In addition, the differences of the Pareto solution on point A and C is larger than that on Point B. It means that the variation of σ_1 has a significant effect on both ends of Pareto curve.

5.2. Crash worthiness problem for side impact

As the second numerical example, the following side impact crash worthiness problem [Sinha, 2007] is considered. The original problem is formulated as RBDO problem by Youn et al. [Youn et al., 2004], where design problem is to minimize the body weight subject to nine reliability constraints such as abdomen load, viscous criteria and rib deflection in terms of nine random variables such as the pillar thickness and the door beam thickness, where the mean values are treated as random variables. The limit state functions are approximated by the response surface method. Sinha [Sinha, 2007] extended this design problem to RBMO by adding the door deflection velocity as the objective function and obtained the Pareto solution by using the multiobjective genetic algorithms, NSGA-II. One of the author applied the STOM to this problem and obtained more accurate Pareto solutions [Kogiso et al., 2014].

The formulation is summarized in Table 4 (a) and (b), where the design variable is denoted as \mathbf{d} that is the mean of random variable $\boldsymbol{\mu}$ and the target reliability is set to $\beta_j^T = 3$. The random variables \mathbf{x} have independent normal distributions.

The standard deviations of x_6 and x_8 were set as unknown to simulate the information uncertainty. These values were determined from the simulation data from the series of random numbers following $N(\mu_6, \sigma_6^2) = N(1.0, 0.03^2)$ and

Table 4 Formulation of crashworthiness problem [Sinha, 2007]

(a) Design variables, side constraints, and covariance in random variables

Name	Variable	Lower bound	Upper bound	σ
Thickness of B-pillar inner (mm)	d_1	0.5	1.5	0.03
Thickness of B-pillar reinforcement (mm)	d_2	0.5	1.5	0.03
Thickness of floor side inner (mm)	d_3	0.5	1.5	0.03
Thickness of cross members (mm)	d_4	0.5	1.5	0.03
Thickness of door beam (mm)	d_5	0.5	1.5	0.03
Thickness of door belt line reinforcement (mm)	d_6	0.5	1.5	unknown
Thickness of roof rail (mm)	d_7	0.5	1.5	0.03
Material yield stress for B-pillar inner (GPa)	d_8	0.192	0.750	unknown
Material yield stress for floor side inner (GPa)	d_9	0.192	0.750	0.006

(b) Objective functions and nine constraints

Name	Upper limit	Formulation
Weight (kg)	f_1	—
Door velocity	f_2	—
Abdomen load (kN)	g_1	1.0
VC upper (m/s)	g_2	0.28
VC middle (m/s)	g_3	0.32
VC lower (m/s)	g_4	0.36
Rib deflection upper (mm)	g_5	28.0
Rib deflection middle (mm)	g_6	30.0
Rib deflection lower (mm)	g_7	38.0
Public symphysis force (kN)	g_8	4.4
B-Pillar velocity (m/s)	g_9	9.9

Table 5 Estimated parameter values

	Sample value	True value	CI* (90%)
μ_6	1.0021	1.0	(0.9769, 1.0298)
σ_6	0.0381	0.03	(0.0298, 0.0962)
μ_8	0.3004	0.3	(0.0253, 0.0366)
σ_8	0.0076	0.006	(0.0066, 0.0216)

CI* means "confidence interval".

Table 6 Set of standard deviation values

Pattern	A	B	C	D
σ_6	0.0298	0.0298	0.0962	0.0962
σ_8	0.0066	0.0216	0.0066	0.0216

Table 7 Relationship between aspiration level and the set of standard deviation values

Asp. level	Type 1 =(20.0,15.5)	Type 2 =(21.5,15.0)	Type 3 =(23.0,14.5)	Type 4 =(24.5,14.0)	Type 5 =(26.0,13.5)
Pattern A	(19.95, 15.44)	(20.93, 14.74)	(22.19, 14.33)	(24.89, 14.03)	(29.33, 13.52)
Pattern B	(19.97, 15.47)	(20.97, 14.76)	(22.22, 14.33)	(24.97, 14.04)	(29.46, 13.52)
Pattern C	(20.25, 15.77)	(21.39, 14.95)	(22.47, 14.39)	(25.37, 14.08)	(30.13, 13.52)
Pattern D	(20.27, 15.80)	(21.40, 14.96)	(22.48, 14.39)	(25.41, 14.08)	(30.19, 13.52)

$N(\mu_8, \sigma_8^2) = N(0.3, 0.006^2)$, where the standard deviations are selected from the original problem [Youn et al., 2004].

$$D_6 = (1.0634 \quad 0.9907 \quad 0.9896 \quad 0.9604 \quad 1.0065)^T \tag{17}$$

$$D_8 = (0.3127 \quad 0.2981 \quad 0.2979 \quad 0.2921 \quad 0.3013)^T \tag{18}$$

First, the distribution parameters of x_6 and x_8 are evaluated by the proposed method. Table 5 lists the sample values, the true value, and the confidence interval of the estimated parameters of x_6 and x_8 . Except for σ_8 , the 90% confidence interval includes the true values.

The Pareto solutions corresponding to the confidence interval of σ_6 and σ_8 are shown in Fig. 6. In this figure, "pattern A" to "D" correspond to the set of the standard deviation values consisting of the upper and the lower bounds of the confidence level shown in Table 6. This figure shows that the Pareto solutions corresponding to each "pattern" are shifted to the right.

Then, to confirm this fact in detail, the Pareto solutions under the five given types of different aspiration levels are compared in Fig. 7. These aspiration levels and the Pareto solutions are listed in Table 7. It is found that change of f_2 is smaller than that of f_1 through Pattern A to D in each type. However, the left side Pareto solutions such as Types 1 and 2 have larger variation of f_2 than the right side solution as Types 4 and 5. Differences of f_1 and f_2 between "pattern A" and "pattern D" are listed in Table 8. It is found that the variations of σ_6 and σ_8 have significant effects on both objective

Table 8 Variations of type 1 to 5

	Δf_1	Δf_2	$\Delta f_1/\Delta f_2$
Type 1	0.3262	0.3524	0.9258
Type 2	0.4672	0.2131	2.1946
Type 3	0.2913	0.0623	4.6753
Type 4	0.5210	0.0446	11.6909
Type 5	0.8522	0.0051	167.9884

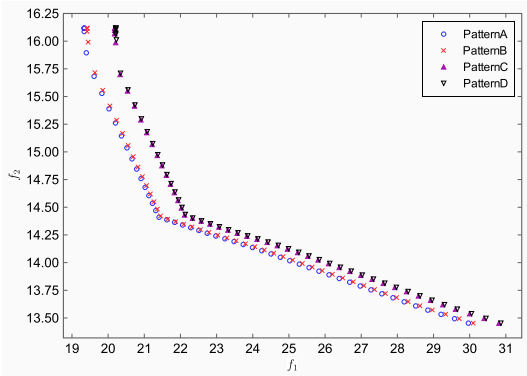


Fig. 6 Confidence interval of Pareto solutions in objective function space in crash worthiness problem

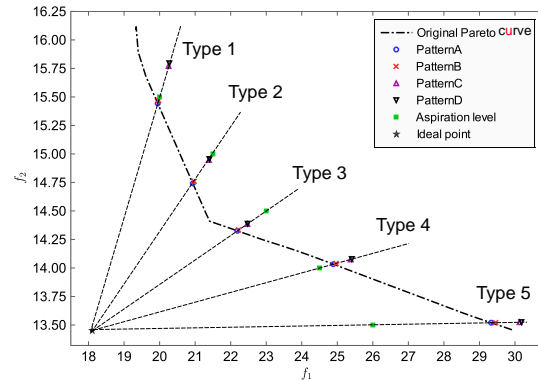


Fig. 7 Ideal point and the aspiration level vs the Pareto solutions under $\beta^T = 3$ in crash worthiness problem

Table 9 Variations of the objective functions ($\beta^T = 3$)

	σ_1			σ_2			σ_3		
	Δf_1	Δf_2	Δf^*	Δf_1	Δf_2	Δf	Δf_1	Δf_2	Δf
Type 1	0.332	0.359	0.489	0.520	0.562	0.766	0.149	0.161	0.219
Type 2	0.510	0.232	0.560	0.666	0.304	0.732	0.228	0.104	0.251
Type 3	0.275	0.059	0.281	0.358	0.077	0.366	0.045	0.010	0.046
Type 4	0.490	0.042	0.492	0.581	0.050	0.583	0.029	0.002	0.029
Type 5	0.803	0.005	0.803	0.984	0.006	0.985	0.000	0.000	0.000
σ_4									
Type 1	0.312	0.337	0.459	0.000	0.000	0.000	0.012	0.013	0.018
Type 2	0.478	0.218	0.526	0.000	0.000	0.000	0.019	0.009	0.021
Type 3	0.256	0.055	0.262	0.000	0.000	0.000	0.003	0.001	0.004
Type 4	0.455	0.039	0.457	0.000	0.000	0.000	0.000	0.000	0.000
Type 5	0.484	0.003	0.484	0.000	0.000	0.000	0.000	0.000	0.000
σ_5									
Type 1	0.312	0.337	0.459	0.000	0.000	0.000	0.012	0.013	0.018
Type 2	0.478	0.218	0.526	0.000	0.000	0.000	0.019	0.009	0.021
Type 3	0.256	0.055	0.262	0.000	0.000	0.000	0.003	0.001	0.004
Type 4	0.455	0.039	0.457	0.000	0.000	0.000	0.000	0.000	0.000
Type 5	0.484	0.003	0.484	0.000	0.000	0.000	0.000	0.000	0.000
σ_6									
Type 1	0.157	0.170	0.232	0.028	0.030	0.041	0.034	0.037	0.051
Type 2	0.242	0.110	0.266	0.043	0.020	0.048	0.053	0.024	0.058
Type 3	0.015	0.003	0.015	0.037	0.008	0.038	0.004	0.001	0.004
Type 4	0.000	0.000	0.000	0.078	0.007	0.078	0.000	0.000	0.000
Type 5	0.000	0.000	0.000	0.127	0.001	0.127	0.000	0.000	0.000
σ_7									
Type 1	0.157	0.170	0.232	0.028	0.030	0.041	0.034	0.037	0.051
Type 2	0.242	0.110	0.266	0.043	0.020	0.048	0.053	0.024	0.058
Type 3	0.015	0.003	0.015	0.037	0.008	0.038	0.004	0.001	0.004
Type 4	0.000	0.000	0.000	0.078	0.007	0.078	0.000	0.000	0.000
Type 5	0.000	0.000	0.000	0.127	0.001	0.127	0.000	0.000	0.000
σ_8									
Type 1	0.157	0.170	0.232	0.028	0.030	0.041	0.034	0.037	0.051
Type 2	0.242	0.110	0.266	0.043	0.020	0.048	0.053	0.024	0.058
Type 3	0.015	0.003	0.015	0.037	0.008	0.038	0.004	0.001	0.004
Type 4	0.000	0.000	0.000	0.078	0.007	0.078	0.000	0.000	0.000
Type 5	0.000	0.000	0.000	0.127	0.001	0.127	0.000	0.000	0.000
σ_9									
Type 1	0.157	0.170	0.232	0.028	0.030	0.041	0.034	0.037	0.051
Type 2	0.242	0.110	0.266	0.043	0.020	0.048	0.053	0.024	0.058
Type 3	0.015	0.003	0.015	0.037	0.008	0.038	0.004	0.001	0.004
Type 4	0.000	0.000	0.000	0.078	0.007	0.078	0.000	0.000	0.000
Type 5	0.000	0.000	0.000	0.127	0.001	0.127	0.000	0.000	0.000

* Δf means the norm of the variation : $\sqrt{\Delta f_1^2 + \Delta f_2^2}$

functions f_1 and f_2 for the left hand side Pareto solutions. On the other hand, at the right hand side of the Pareto solutions, the variations of σ_6 and σ_8 have effect only on f_1 .

5.3. Investigation of effect of confidence interval on the Pareto solution

In the above section, the effect of information uncertainty is investigated by applying the parameter estimation method. It is shown that σ_6 and σ_8 have a large effect on the Pareto solution. Then, in this section, we investigate the most effective random variable. The assumption is made that the confidence intervals from σ_1 to σ_7 and from σ_8 to σ_9 are equal to σ_6 and σ_8 , respectively, as listed in Table 5, because they have the same probabilistic parameter in the original problem [Youn et al., 2004].

First, the target reliability is set to $\beta_j^T=3$ (this value is the same as in the above section). Then, the Pareto solutions corresponding to the confidence interval of $\sigma_i(i = 1, \dots, 9)$ are evaluated in Table 9. Except for type 1, the variation of each σ has a greater effect on Δf_1 than on Δf_2 . In using type 1, i.e. the left hand side of the Pareto solution, each σ has an effect on either f_1 or f_2 . $\sigma_1, \sigma_2,$ and σ_4 have a significant effect on each objective function. On the other hand, σ_5 has absolutely no effect on any of the objective functions. Their confidence intervals are compared in Fig. 8 (a) to (d). In

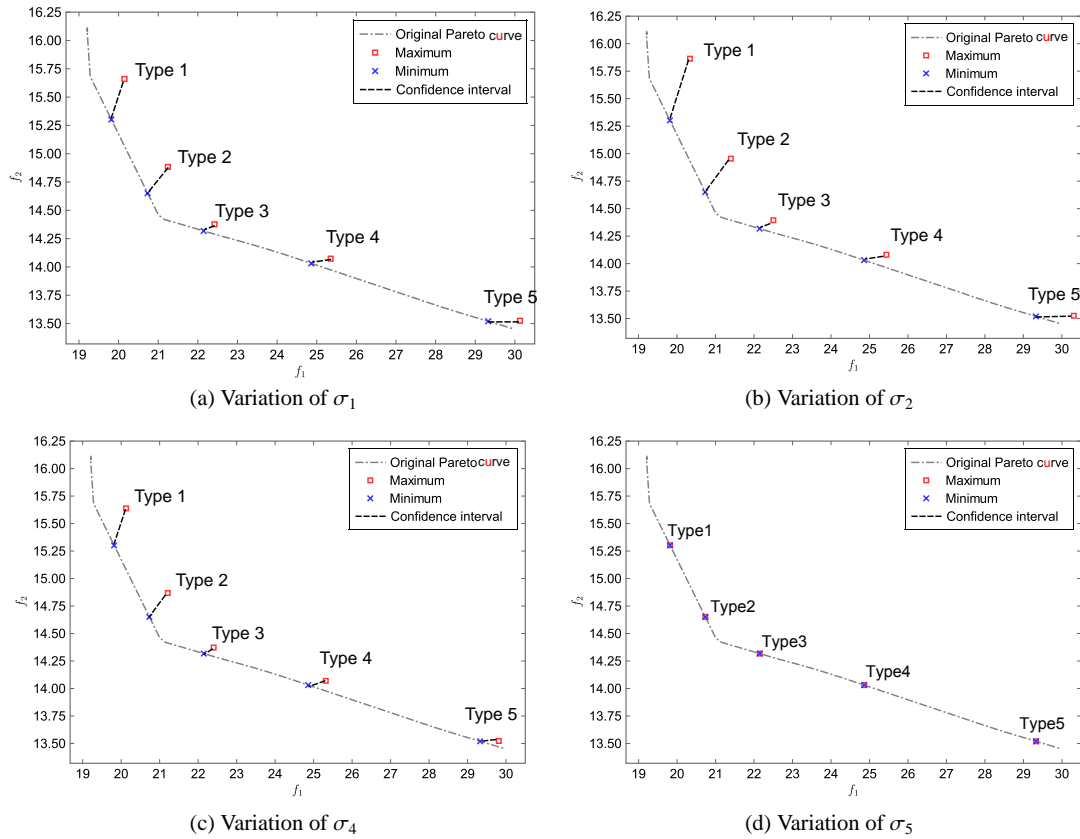


Fig. 8 Confidence interval of Pareto solution corresponding to the variation of variance in objective function space ($\beta^T = 3$)

Table 10 Variations of objective functions ($\beta^T = 6$)

	σ_1			σ_2			σ_3		
	Δf_1	Δf_2	Δf^*	Δf_1	Δf_2	Δf	Δf_1	Δf_2	Δf
Type 1	1.656	0.237	1.673	2.157	0.115	2.160	0.555	0.225	0.599
Type 2	0.956	0.435	1.050	1.248	0.569	1.371	0.521	0.238	0.573
Type 3	1.076	0.230	1.100	1.444	0.309	1.477	0.529	0.113	0.540
Type 4	0.988	0.085	0.992	1.245	0.106	1.249	0.099	0.008	0.100
Type 5	1.613	0.010	1.613	2.111	0.013	2.111	0.125	0.001	0.125
	σ_4			σ_5			σ_6		
Type 1	1.534	0.219	1.549	0.006	0.006	0.008	0.006	0.006	0.008
Type 2	0.955	0.435	1.049	0.038	0.018	0.042	0.038	0.018	0.042
Type 3	1.074	0.230	1.098	0.002	0.000	0.002	0.002	0.000	0.002
Type 4	0.896	0.077	0.899	0.000	0.000	0.000	0.000	0.000	0.000
Type 5	1.202	0.007	1.202	0.000	0.000	0.000	0.000	0.000	0.000
	σ_7			σ_8			σ_9		
Type 1	0.492	0.189	0.527	0.142	0.153	0.209	0.010	0.011	0.014
Type 2	0.478	0.218	0.526	0.084	0.038	0.093	0.107	0.049	0.117
Type 3	0.474	0.101	0.485	0.052	0.011	0.053	0.009	0.002	0.009
Type 4	0.023	0.002	0.023	0.189	0.016	0.190	0.000	0.000	0.000
Type 5	0.228	0.001	0.228	0.234	0.001	0.234	0.000	0.000	0.000

these figures, “minimum” and “maximum” indicate the lower and the upper side of the confidence interval, respectively.

Next, the target reliability is set as $\beta^T = 6$. The Pareto solutions corresponding to the confidence interval are compared in Table 10, and the Pareto solutions corresponding to variations of σ_1 , σ_2 , σ_4 , and σ_5 are shown in Fig. 9 (a) to (d), respectively. Table 10 lists the variations of objective functions in case of $\beta^T = 6$. In comparison with Table 9, it is found that the variations of the objective functions become larger, as the target reliability is larger.

Figure 9 (a) to (d) illustrate the confidence intervals in the case of $\beta^T = 6$. The variations of σ_1 , σ_2 , and σ_4 have a significant effect and the variation of σ_5 has absolutely no effect.

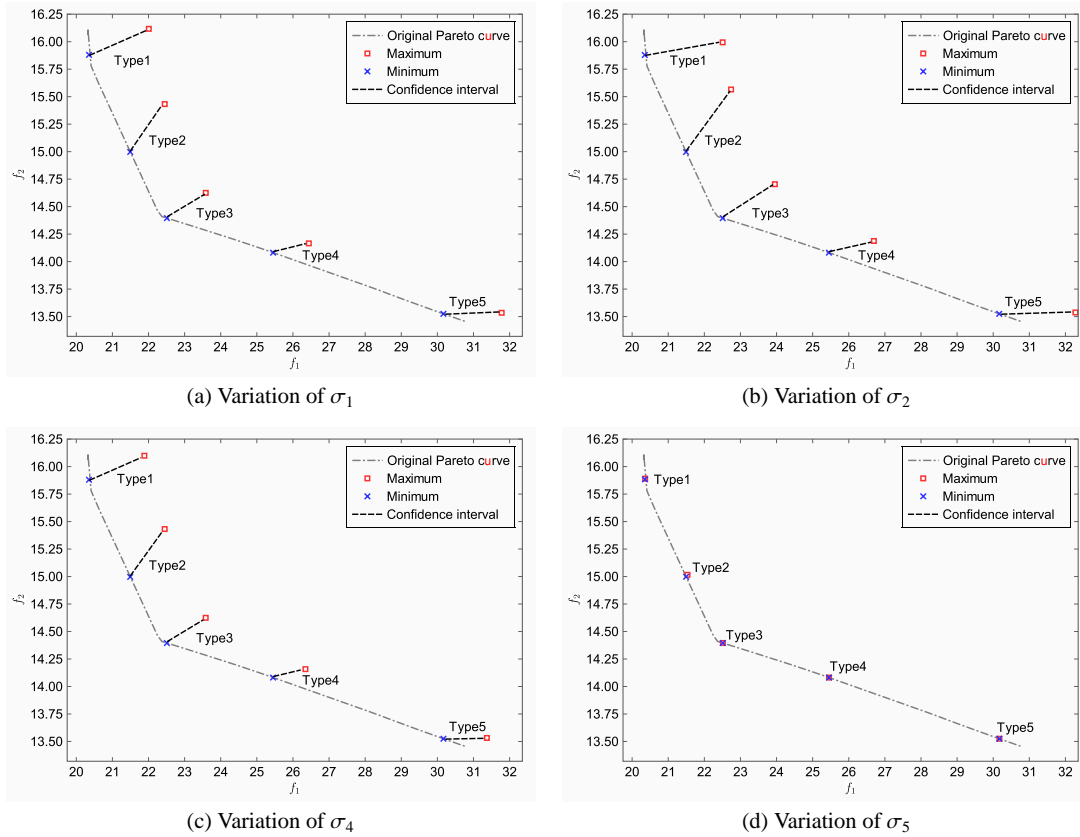


Fig. 9 Confidence interval of Pareto solution corresponding to the variation of variance in objective function space ($\beta^T = 6$)

6. Conclusion

This paper proposes an evaluation method to estimate the effect of the information uncertainty on the reliability-based multiobjective optimum design. Our proposed method is applied to the case when the probabilistic parameters are not estimated with high accuracy due to lack of sample numbers. In such a case, the uncertainties in the distribution parameters, that is the variations of the sample mean and sample variance, are not negligible. Our proposed method is developed to improve the estimation accuracy based on Bayes' theorem, where the distribution parameter uncertainty is evaluated by using the confidence interval. Through numerical examples, the proposed sampling method is shown to be more efficient than conventional Gibbs sampling.

Then, the effects of the uncertainty in the parameters on reliability-based optimum design are evaluated through the confidence interval of Pareto solutions of RBMO. This confidence interval is evaluated by obtaining the Pareto solutions after setting the distribution parameters as the bound values of the confidence interval of the random variable. Through this study, the variation of the Pareto solution considering information uncertainty in addition to physical uncertainty is evaluated by the integration of STOM and the parameter estimation method proposed in our previous paper [Ito and Kogiso, 2015].

To solve the RBMO problem efficiently, this study adopts STOM for evaluating the Pareto solutions. STOM is useful to obtain the single Pareto solution corresponding to an aspiration level. This advantage is appropriate to investigate the effect of the distribution parameter uncertainty on the confidence interval of each Pareto solution as shown in numerical examples.

Acknowledgments

This study is partially supported by the Japan Society for the Promotion of Science (JSPS) Grants-in-Aid for Scientific Research (KAKENHI) on Grant Number 16K13738.

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