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NATURAL CONVECTION OF PARAMAGNETIC FLUID BETWEEN PARALLEL PLATES UNDER STRONG MAGNETIC FIELD

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ABSTRACT The paramagnetic fluid has the positive magnetic susceptibility which depends on the temperature. This suggests that the heat and fluid flow of air can be controlled by a magnetic field. This phenomenon is called magnetothermal convection. In this study, the effect of the magnetothermal force is numerically investigated on the natural convection between vertical heated parallel plates under the gravity field. The heat and fluid flow is simultaneously solved by the lattice Boltzmann method (LBM). For the evaluation of the external force on LBM, the validation is firstly carried out for the natural convection in a square cavity. It is confirmed that employed scheme has a comparable validity to the reference data. In the computation for the natural convection between the parallel plates, it is found that, because the hotter fluid receives weaker magnetic force, the magnetothermal force is expressed as a repelling force. This force is directed to perpendicular to the flow direction, thus the hotter fluid is taken away to the bulk region, and the colder fluid is attracted toward the magnet. Therefore, the thermal boundary layer at the magnet location is affected by the magnetothermal force. The local heat transfer enhancement is also discussed with the different wall heat flux and magnet location.

INTRODUCTION

The magnetic susceptibility is one of physical properties of materials. The material of positive susceptibility is attracted to the magnet. The paramagnetic materials including oxygen and air, have the positive susceptibility with much smaller magnitude compared with the ferromagnetic ones. Another characteristic of the paramagnetic materials is that the magnetic susceptibility depends on the inverse of the absolute temperature. Therefore, colder paramagnetic material can be attracted stronger to the magnet than hotter one. This has been known in the past, and applied to the gas sensors. However, the application to the engineering process was limited due to its weak susceptibility. Since the emergence of superconducting magnet, various new findings are found such as the levitation of a water droplet by Ikezoe et al. [1998], a nitrogen jet by Wakayama et al. [1991], magnetoarchimedes effect by Uetake et al. [2000]. The application of the strong magnetic field has been also discussed such to the protein crystallization by Maki et al. [2004], convection enhancement / suppression by Wrobel et al. [2012]. In terms of the convection control, the magnetothermal force is comparative to the natural convection suggested by Bednarz et al. [2004]. However, to enhance its effect, it is preferable to introduce small flow domain because the gap of the magnet can be narrower. In this study, the natural convection between parallel plates whose gap is in millimeter order is targeted, and the effect of the magnetothermal force on the heat transfer is numerically investigated.

COMPUTATIONAL METHOD

Magnetothermal Lattice Boltzmann Method The heat and fluid flow is simultaneously solved by the lattice Boltzmann method (LBM). In the present study, two-dimensional nine-discrete velocity

(D2Q9) model is employed, which is illustrated in Fig. 1. (CUDA Fortran code is developed by ourselves, thus the discrete velocity numbering starts from 1.) Two distribution functions are respectively employed for the density and thermal energy. The time evolution equations of both distribution functions at the lattice site *r* and time *t* are single-relaxation-time BGK models as below.

$$f_\alpha(r+e_\alpha\delta t, t+\delta t)= f_\alpha(r,t)-\frac{\delta t}{\tau_f+0.5\delta t}\left(f_\alpha(r,t)-f_\alpha^{eq}(r,t)\right)+\frac{\tau_f}{\tau_f+0.5\delta_t}F_\alpha \quad (1)$$

$$g_\alpha(r+e\delta t, t+\delta t)= g_\alpha(r,t)-\frac{\delta t}{\tau_g+0.5\delta t}\left(g_\alpha(r,t)-g_\alpha^{eq}(r,t)\right) \quad (2)$$

The superscript eq is the equilibrium state. The discrete vector is e_α. τ_f and τ_g are the relaxation times. In the density distribution function (Eq.(1)), external forces due to buoyancy f_g and magnetothermal forces f_m are considered. The expression of the external force is referred to He et al. [1998].

$$F_\alpha=\frac{(f_g+f_m)\times(e_\alpha-u)}{RT_0}f_\alpha^{eq} \quad (3)$$

Where,

$$\boldsymbol{f}_g=-\beta\rho_0(T-T_0)g \quad (4)$$

$$\boldsymbol{f}_m=-\frac{\beta\rho_0\chi_0}{\mu_m}(T-T_0)\nabla\boldsymbol{b}^2 \quad (5)$$

These suggest followings. The buoyancy force works due to temperature difference (Boussinesq approximation). The magnetothermal force depends on the magnetic susceptibility χ, the local temperature, and the local gradient of the squared magnetic induction. The derivation of the magnetothermal force is referred from Kaneda et al. [2015].
The macroscopic parameters are related as follows.

$$\rho=\sum_\alpha f_\alpha,\ \rho u=\sum_\alpha f_\alpha e_\alpha+0.5F_\alpha,\ p=\frac{1}{3}\rho,\ \nu=\frac{1}{3}\tau_f,\ T=\sum_\alpha g_\alpha(r,t),\ D=\frac{1}{3}\tau_g \quad (6\text{-}11)$$

Where, *D* is the thermal diffusivity.

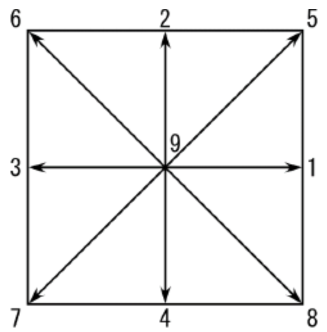


Fig. 1 D2Q9 model.

Magnetic Field The magnetic field induced by a block magnet can be referred from a textbook of the electromagnetism. In the present study, a cusp-shaped magnetic field is estimated by the overlapping of magnetic field from each magnet.

RESULTS AND DISCUSSION

Evaluation of External Force on Natural Convection In the present study, the consideration of the external force is referred to He et al.[1998] as Eq. (3). Firstly, the validation exercise is carried out for the natural convection in a square cavity. In the LBM, the Prandtl number is adjusted by the relaxation times since they are related to the kinematic viscosity and the thermal diffusivity, respectively. In this section, the Prandtl number is presumed to 0.71. The Rayleigh number can be set by giving the corresponding value to the gravity acceleration and volumetric expansion coefficient because the dimensionless temperature is respectively given as boundary conditions on hot and cold walls, and other dimensionless physical properties are fixed due to the relaxation times. The boundary conditions are given by the half-way bounce-back manner. The node number for the fluid domain is 200^2, which is sufficient for the grid independent result.

The resulted average Nusselt number on the hot wall is listed in Table 1. The results depend on the combination of the relaxation time even though the estimated Prandtl number is identical. After some numerical tests, the appropriate combination of the relaxation times is confirmed, which are $\tau_f = 0.08875$ and $\tau_g = 0.1250$. For the comparison, well-known benchmarks by Hortmann et al. [1990] and de Vahl Davis [1983] are referred. It is confirmed that the present scheme has sufficient validation. For further confirmation, promising scheme in LBM by Guo et al. [2002] is also tested. The deviation is less than the single-precision order.

Table 1
Average Nusselt Number on Heated Wall in a Square Cavity

	Rayleigh number		
	10^4	10^5	10^6
Present model	2.245	4.522	8.821
Hortmann et al. [1990]	2.245	4.521	8.825
de Vahl Davis [1983]	2.238	4.509	8.817

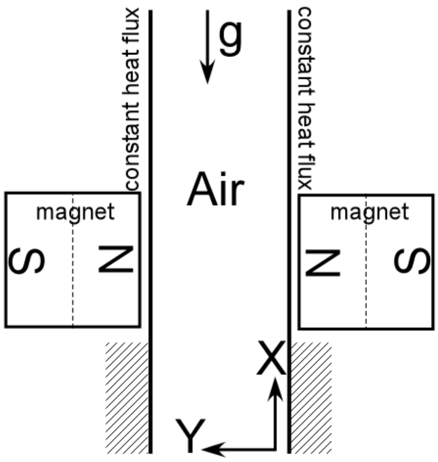


Fig. 2. Schematic model for natural and magnetothermal convections.

Natural and Magnetothermal Convections

Heat and fluid flow. Fig. 2 shows a schematic model for the computation of magnetothermal and natural convections. A pair of parallel plates is set vertically. The plates are thermally insulated at the bottom region (X<50) and heated at a constant heat flux (X≥50). Air is presumed as a working fluid (Pr = 0.71). Two block magnets are facing at a same elevation, which induces the cusp-shaped magnetic field. The computational nodes for a fluid domain is 261 × 81 (Aspect ratio is thus 3.22). At the bottom end, the temperature is set 1.0, and the top end is presumed thermally insulated. The elevation of the magnets can be shifted arbitrarily. Because the constant heat flux is presumed, the modified Rayleigh number is employed for the computational parameter. Another nondimensional parameter γ is introduced for the magnetic induction. The derivation of γ can be found such in Kaneda et al. [2002].

$$Ra^* = g\beta\dot{q}\ell^4 / \lambda D \nu \tag{12}$$
$$\gamma = \chi_m b^2 / (\rho g \mu_m \ell) \tag{13}$$

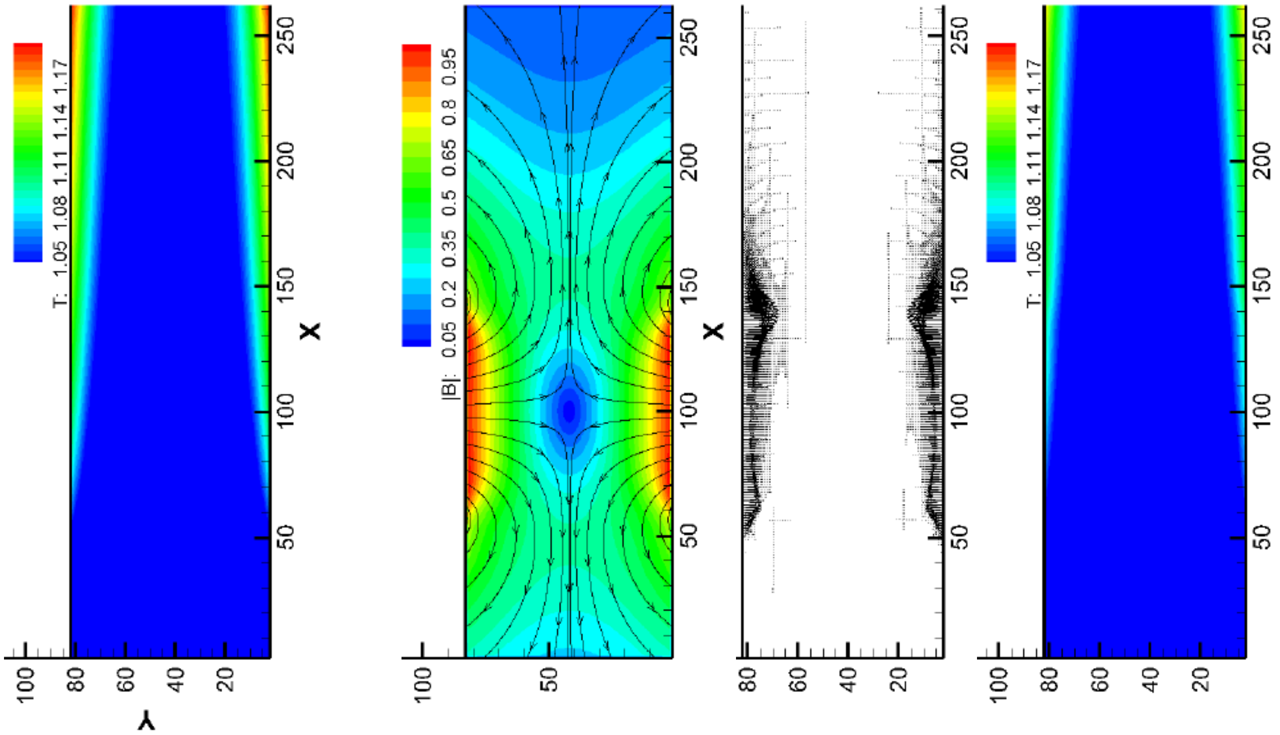
The magnetic field, the magnetothermal force, streamlines and the isothermal contours are shown in Fig. 3 for the case of Ra* = 10^4 and γ = 50. The size of the block magnet is 80× 80, and its center is at X = 100 (Thus the magnet covers 60 ≤ X ≤ 140). The corresponding magnetic induction at the magnet surface is 0.88 Tesla and the given heat flux is 3.3×10^4 [W/m^2] at the plate gap of 4mm for air. Because the magnetothermal force is due to the local temperature difference from the reference temperature (= inlet temperature) and the local magnetic induction, the resulted force becomes a repelling force toward the bulk region. The force becomes stronger at higher elevation, thus the flow is pulled away from the wall in a vicinity of the magnet as the local temperature increases. Due to the mass conservation law and upstream buoyant flow near the wall, the fluid flow is wound at the magnet location and the thermal boundary layer becomes thin at lower half of the magnet area. Consequently, the growth of the thermal boundary layer is delayed along the wall.

Local heat transfer. As shown in the Fig. 3, thermal boundary layer is affected by the magnetothermal force. Therefore, the effect of the magnetothermal force on the heat transfer from the wall is investigated. Two kinds of Ra* are examined with two magnet elevations. For the magnet elevations, one is that the magnet covers 10≤X≤80 (case 1). In this case, bottom edge of the magnet is out of the heated region. The another is for 60≤X≤140 (case 2) which corresponds to the case in Fig. 3. The results are summarized in Fig. 4. The local Nusselt number along the heated region (X≥50) is plotted.

Interestingly, the heat transfer from the wall is enhanced in all cases with magnetic field. Moreover, in case at Ra* = 10^3 (Fig. 4(a)), the difference depending on the magnet location is quite small. This implies that the heat transfer enhancement is driven at the upper half of the magnet, since the bottom half is not in the heated region in case 1. In addition, the fluid flow is not wound so much in these case.

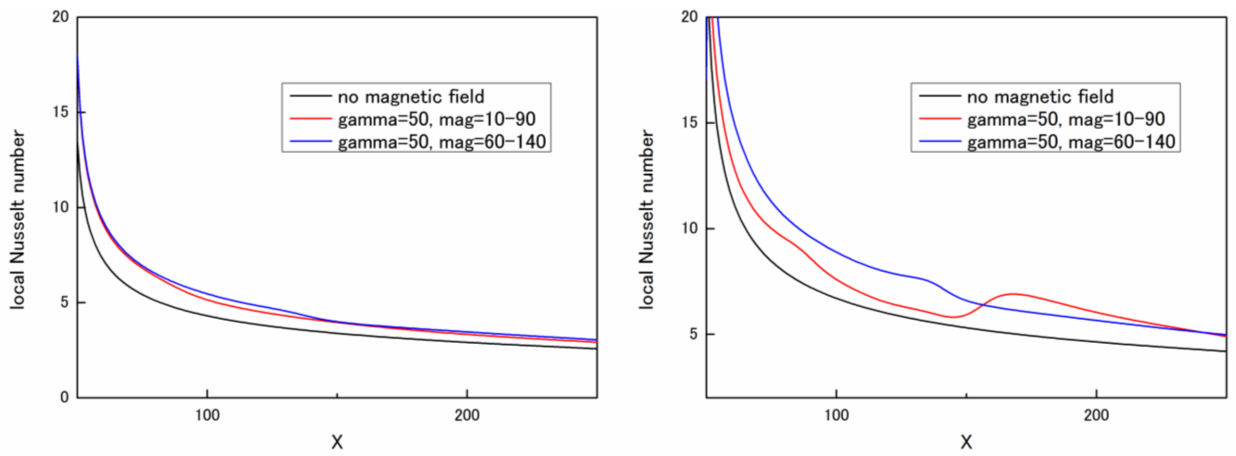
At larger heat flux at Ra* = 10^4, the dependence of the magnet location becomes remarkable. In the case 1, the heat transfer is enhanced at the vicinity of the magnet. At the upper magnet edge, the enhancement is reduced. The enhancement occurs again far away from the magnet. This can be understood that, a virtual rib effect occurs at the top magnet edge location. The virtual rib effect is reported for the forced convection case by Kaneda et al. [2016]. As shown in Fig. 3(c), the repelling force becomes maximum at the top magnet edge. Once the flow passes over the rib, the flow reattaches at the wall. The heat transfer is enhanced at the reattaching point. In the case 2, this rib effect is not strong compared with case 1, which decreases the reattachment effect.

These results are quite interesting, because the effect of the magnetothermal force on the natural convection becomes different from that on the forced convection



(a) no magnetic field (b) magnetic field (c) magnetothermal force (d) isotherms

Fig. 3. Magnetothermal force overlapping on the natural convection between a parallel plates at Pr = 0.71, Ra* = 10^4, and $\gamma = 50$. The magnet area is X = 60-140.



(a) Ra* = 10^3 (b) Ra* = 10^4

Fig. 4. Local Nusselt number along the heated wall

CONCLUSION

The magnetothermal convection of air is considered in the natural convection between parallel plates. The computations are carried out by the lattice Boltzmann method. Firstly, the appropriate relaxation times are obtained by the numerical test in the natural convection inside a square cavity. For the computation of the parallel plates, it is found that the thermal boundary becomes thin on the magnet location because the hotter fluid is repelled from the wall near the top magnet edge (and the colder fluid is attracted). By the estimation of the local Nusselt number, the magnetothermal force enhances the heat transfer of the natural convection induced by heated parallel vertical wall. This implies that the application of the strong permanent magnet is effective to enhance the heat transfer of the small-sized heat sinks, where the natural convective heat transfer is dominant.

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