Safety Margins for Reliability Analysis of Frame Structures

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(Received November 15, 1983)

The new method is proposed in this paper for generating safety margins of general frame structures, taking account of interaction of the applied loads on yielding of the sections and structural failure defined as production of large nodal displacements due to the plastic collapsing. The plasticity condition of the sections is approximated by a linear surface and the matrix method is applied to formulate the safety margins as linear combinations of the strengths of the elements and the applied loads, which greatly facilitates reliability analysis of the frame structures under any loading conditions.

1. Introduction

The early studies on reliability analysis of frame structures were focussed on estimation of reliability by evaluating its lower and upper bounds for given modes of failure[1~3]. It is difficult in practice to specify the relevant modes of failure and their equations *a priori* for large structures with high degrees of redundancy. Consequently, identification of stochastically significant failure modes is recognized to be an essential step to be done for reliability assessment of structural systems. Researches for automatically generating mode equations of truss or frame structures were initiated in the case where failure of structural elements was governed simply by axial forces or bending moments.[4~11] However, failure criteria under combined loads have not been fully applied to reliability analysis of general frame structures.[12]

This paper is concerned with a new method of generating safety margins for general frame structures by taking account of interaction of the applied load effects on an yielding section. For the purpose, the plasticity condition of a structural element is at first approximated by a linear surface, and then the corresponding reduced stiffness matrices and equivalent nodal forces representing the residual strengths of the yielded elements are derived for the plastic analysis, by using a plastic deformation theory. Finally, safety margins for reliability analysis are formulated by using a matrix method.

2. Elastic-Plastic Analysis of Frame Structures

2.1 Basic assumptions

The following assumptions are made, concerning frame structures to be considered:

(1) Consider a frame structure whose elements are uniform and homogeneous and to which only concentrated loads are applied. In such a frame structure,

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critical sections where plastic hinges may form are the joints of the elements and the places at which the concentrated loads are applied. Consequently, those potential plastic hinged sections are taken as the ends of the elements to facilitate structural analysis.

(2) Yielding of a section occurs when the yield function F_k is equal to zero, that is, $F_k=0$. Further, the yield function F_k is determined by the dimension and yield stress of the element as well as the applied internal forces $\boldsymbol{X}_t$.

(3) Mechanical behaviours of materials are perfectly elastic-plastic. That is, the plastic hinged sections follow the plastic deformation theory, and the other section behaves elastically.

2.2 Plasticity condition

Let $\boldsymbol{X}_t$ and $\boldsymbol{\delta}_t$ denote the nodal force and displacement vectors of the unit element i, j, *e.g.*, the element number t in the local coordinate system shown in Fig. 1.

From the assumption (1), the bending moment varies linearly from node i to j. It follows that the maximum bending moment of the unit element occurs at one or both of the nodes. Eventually, the yielding occurs at one or both of the ends of the unit elements when the plasticity condition $F_k=0$ $(k=i, j)$ is satisfied. In case where the interaction at the yield section is not taken into account, this condition is simple.[4~11] In order to overcome the difficulties encountered in failure analysis[13] considering the interaction effect of the internal forces upon the plasticity condition, the yield function is approximated by a linearized surface as given in the following form:

$$F_k=R_k-\boldsymbol{C}_k^T\boldsymbol{X}_t=0 \qquad (k=i, j) \tag{1}$$

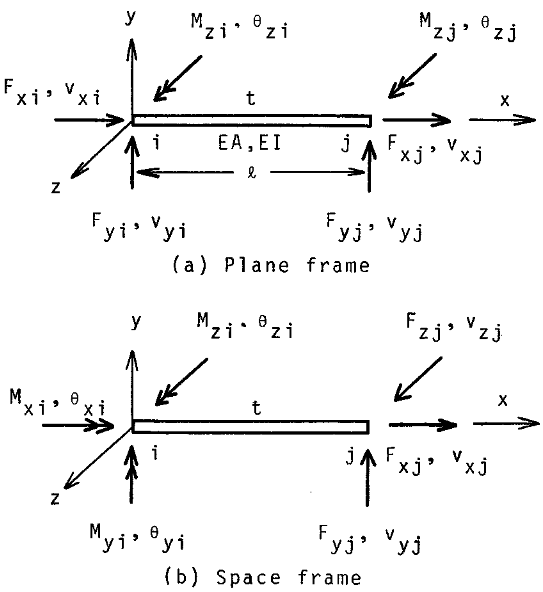

Fig. 1 Nodal forces and nodal displacements.

where R_k : strength of the element end k,
$\boldsymbol{C}_k^T$: factor determined by the dimension of the element.

Several examples of the plasticity conditions based on the above approximation are given for explanation:

(1) Plane frames where the interaction of the bending moment and axial force upon the plasticity condition is taken into account and when a fully plastic moment is taken as the reference strength (see Fig. 2):

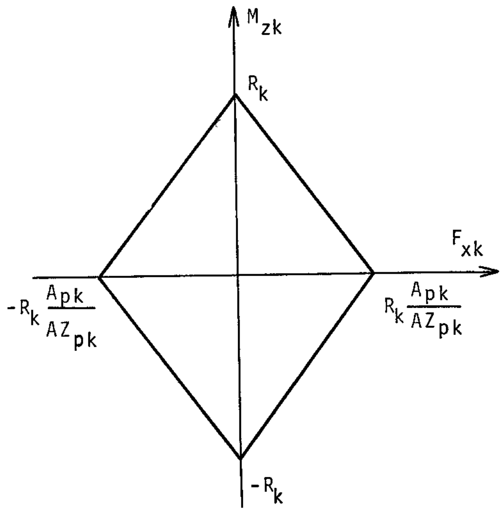

Fig. 2 Linearized plasticity condition considering the interation of the bending moment and axial force.

$$R_k=\sigma_{yk}AZ_{pk} \quad \text{(a)}$$

$$\boldsymbol{C}_i^T=(AZ_{pi}/A_{pi}\ \text{sign}\ (F_{xi}),\ 0,\ \text{sign}\ (M_{zi}),\ 0,\ 0,\ 0) \quad \text{(b)}$$

$$\boldsymbol{C}_j^T=(0,\ 0,\ 0,\ AZ_{pj}/A_{pj}\ \text{sign}\ (F_{xj}),\ 0,\ \text{sign}\ (M_{zj})) \quad \text{(c)}$$

where σ_{yk} : yield stress
AZ_{pk}: plastic section modulus
A_{pk} : area of an element section
sign (•): sign of (•)

$$\boldsymbol{X}_t=(F_{xi},\ F_{yi},\ M_{zi},\ F_{xj},\ F_{yj},\ M_{zj})^T \quad \text{(d)}$$

In particular, the well-known plasticity condition subjected solely to the bending moment is obtained by making the first term of $\boldsymbol{C}_i^T$ and the fourth term of $\boldsymbol{C}_j^T$ equal to zero.

(2) Space frames where interaction of the bending moments and an axial force is considered and when a fully plastic moment about the z axis is taken as the reference strength:

$$R_k=\sigma_{yk}AZ_{zpk} \quad \text{(e)}$$

$$\boldsymbol{C}_i^T=(AZ_{zpi}/A_{pi}\ \mathrm{sign}\ (F_{xi}),\ 0,\ 0,\ 0,\ AZ_{zpi}/AZ_{ypi}\ \mathrm{sign}\ (M_{yi}),\ \mathrm{sign}\ (M_{zi}),\ 0,\ 0,\ 0,\ 0,\ 0,\ 0) \quad \text{(f)}$$

$$\boldsymbol{C}_j^T=(0,\ 0,\ 0,\ 0,\ 0,\ 0,\ AZ_{zpj}/A_{pj}\ \mathrm{sign}\ (F_{xj}),\ 0,\ 0,\ 0\ AZ_{zpj}/AZ_{ypj}\ \mathrm{sign}\ (M_{yj}),\ \mathrm{sign}\ (M_{zj})) \quad \text{(g)}$$

where AZ_{zpk}, AZ_{ypk}: plastic section modulus about the z and y axes, respectively

$$\boldsymbol{X}_t=(F_{xi},\ F_{yi},\ F_{zi},\ M_{xi},\ M_{yi},\ M_{zi},\ F_{xj},\ F_{yj},\ F_{zj},\ M_{xj},\ M_{yj},\ M_{zj})^T \quad \text{(h)}$$

2.3 Derivation of reduced stiffness matrices and equivalent nodal forces

When an element remains elastic, the relation between the nodal force vector $\boldsymbol{X}_t$ and displacement vector $\boldsymbol{\delta}_t$ of an element is written as

$$\boldsymbol{X}_t=\boldsymbol{k}_t\boldsymbol{\delta}_t \quad (2)$$

where $\boldsymbol{k}_t$: elastic element stiffness matrix

After a section of the element has yielded, *i.e.*, the plasticity condition $F_k=0$ has attained, the relation between $\boldsymbol{X}_t$ and $\boldsymbol{\delta}_t$ will be derived in the following.

The total displacement $\boldsymbol{\delta}_t$ of the element is assumed to consist of an elastic displacement $\boldsymbol{\delta}_t^e$ and a plastic displacement $\boldsymbol{\delta}_t^p$, *i.e.*,

$$\boldsymbol{\delta}_t=\boldsymbol{\delta}_t^e+\boldsymbol{\delta}_t^p=\boldsymbol{\delta}_t^e+\boldsymbol{\delta}_i^p+\boldsymbol{\delta}_j^p \quad (3)$$

Based on the plastic deformation theory, the plastic deformation is expressed in the form:

$$\left.\begin{aligned}\boldsymbol{\delta}_i^p&=\lambda_i\frac{\partial F_i}{\partial \boldsymbol{X}_t}=-\lambda_i\boldsymbol{C}_i\\ \boldsymbol{\delta}_j^p&=\lambda_j\frac{\partial F_j}{\partial \boldsymbol{X}_t}=-\lambda_j\boldsymbol{C}_j\end{aligned}\right\} \quad (4)$$

where λ_i and λ_j are factors to indicate the magnitude of plastic deformation. For example, when section i (j) is elastic, $\lambda_i=0$ ($\lambda_j=0$).

Nodal force $\boldsymbol{X}_t$ is expressed as

$$\boldsymbol{X}_t=\boldsymbol{k}_t\boldsymbol{\delta}_t^e=\boldsymbol{k}_t(\boldsymbol{\delta}_t-\boldsymbol{\delta}_t^p) \quad (5)$$

Substituting Eqs. (3) and (4) into Eq. (5) gives

$$\boldsymbol{X}_t=\boldsymbol{k}_t\boldsymbol{\delta}_t+\lambda_i\boldsymbol{k}_t\boldsymbol{C}_i+\lambda_j\boldsymbol{k}_t\boldsymbol{C}_j \quad (6)$$

Substituting Eq. (6) into Eq. (1) reduces to:

$$R_i-\boldsymbol{C}_i^T(\boldsymbol{k}_t\boldsymbol{\delta}_t+\lambda_i\boldsymbol{k}_t\boldsymbol{C}_i+\lambda_j\boldsymbol{k}_t\boldsymbol{C}_j)=0 \quad (7\text{–}1)$$

$$R_j-\boldsymbol{C}_j^T(\boldsymbol{k}_t\boldsymbol{\delta}_t+\lambda_i\boldsymbol{k}_t\boldsymbol{C}_i+\lambda_j\boldsymbol{k}_t\boldsymbol{C}_j)=0 \quad (7\text{–}2)$$

From Eqs. (7), the relation between λ_i, λ_j and $\boldsymbol{\delta}_t$ is derived. By substituting the resulting relation into Eq. (6), the following equation is obtained:

$$\boldsymbol{X}_t=\boldsymbol{k}_t^{(p)}\boldsymbol{\delta}_t+\bar{\boldsymbol{X}}_t^{(p)} \quad (8)$$

where $k_t^{(p)}$: reduced element stiffness matrix
$\bar{X}_t^{(p)}$: equivalent nodal force vector

The explicit forms of $k_t^{(p)}$, $\bar{X}_t^{(p)}$ and λ_k are expressed as follows:

(1) In case of an elastic element:

$$\left.\begin{aligned} &\lambda_i=\lambda_j=0\\ &k_t^{(p)}=k_t\\ &\bar{X}_t^{(p)}=0 \end{aligned}\right\} \qquad (9\text{-}1)$$

(2) In case of failure at the left-hand end:

$$\left.\begin{aligned} &\lambda_i=(R_i-C_i^Tk_t\delta_t)/(C_i^Tk_tC_i), \qquad \lambda_j=0\\ &k_t^{(p)}(=k_t^L)=k_t-k_tC_iC_i^Tk_t/(C_i^Tk_tC_i)\\ &\bar{X}_t^{(p)}(=\bar{X}_t^L)=R_ik_tC_i/(C_i^Tk_tC_i) \end{aligned}\right\} \qquad (9\text{-}2)$$

(3) In case of failure at the right-hand end:

$$\left.\begin{aligned} &\lambda_i=0, \quad \lambda_j=(R_j-C_j^Tk_t\delta_t)/(C_j^Tk_tC_j)\\ &k_t^{(p)}(=k_t^R)=k_t-k_tC_jC_j^Tk_t/(C_j^Tk_tC_j)\\ &\bar{X}_t^{(p)}(=\bar{X}_t^R)=R_jk_tC_j/(C_j^Tk_tC_j) \end{aligned}\right\} \qquad (9\text{-}3)$$

(4) In case of failure at the both ends:

$$\left.\begin{aligned} &\begin{Bmatrix}\lambda_i\\ \lambda_j\end{Bmatrix}=-[G^{-1}][H]\delta_t+[G^{-1}]\begin{Bmatrix}R_i\\ R_j\end{Bmatrix}\\ &\qquad [G^{-1}]=\begin{bmatrix}C_i^Tk_tC_j & C_i^Tk_tC_j\\ C_j^Tk_tC_i & C_j^Tk_tC_j\end{bmatrix}^{-1}, \quad [H]=\begin{bmatrix}C_i^Tk_t\\ C_j^Tk_t\end{bmatrix}\\ &k_t^{(p)}(=k_t^{LR})=k_t-[H]^T[G^{-1}][H]\\ &\bar{X}_t^{(p)}(=\bar{X}_t^{LR})=[H]^T[G^{-1}]\begin{Bmatrix}R_i\\ R_j\end{Bmatrix} \end{aligned}\right\} \qquad (9\text{-}4)$$

The reduced element stiffness matrix $k_t^{(p)}$ and the equivalent nodal force vector $\bar{X}_t^{(p)}$ are analytically derived from Eqs. (a), (b), and (c) and they are given in Fig. 3 for the plane frame in which the interaction of the bending moment and axial force upon the plasticity condition is considered. Moreover, it should be noted that the following fact is observed from the results in Fig. 3. By taking $C^L=C^R=0$, the reduced element stiffness matrix and the equivalent force vector for a plane frame subjected to the bending moment alone are obtained, and those for a truss structure subjected only to the axial force are given by putting, $C^R\to\infty$ and $R_j/(C^R\cdot l)\to R_j(=\sigma_{yj}A_{pj})$.

3. Generation of Safety Margins

Consider a frame structure with n elements and at most ml loads (m: degree of freedom of a node) applied to its l nodes. Let the left- and right-hand ends

$$\mathbf{k}_t^{(p)} = \mathbf{k}_t^{L}$$

$$\begin{bmatrix}
\frac{EA}{\ell}\cdot\frac{1}{1+\square} & -\frac{EA}{\ell}\cdot\frac{3C^L}{2(1+\square)} & -EA\cdot\frac{C^L}{1+\square} & -\frac{EA}{\ell}\cdot\frac{1}{1+\square} & \frac{EA}{\ell}\cdot\frac{3C^L}{2(1+\square)} & -EA\cdot\frac{C^L}{2(1+\square)} \\
 & \frac{3EI}{\ell^3}\cdot\frac{1+4\square}{1+\square} & \frac{6EI}{\ell^2}\cdot\frac{\square}{1+\square} & \frac{EA}{\ell}\cdot\frac{3C^L}{2(1+\square)} & -\frac{3EI}{\ell^3}\cdot\frac{1+4\square}{1+\square} & \frac{3EI}{\ell^2}\cdot\frac{1+2\square}{1+\square} \\
 & & \frac{4EI}{\ell}\cdot\frac{\square}{1+\square} & EA\cdot\frac{C^L}{1+\square} & -\frac{6EI}{\ell^2}\cdot\frac{\square}{1+\square} & \frac{2EI}{\ell}\cdot\frac{\square}{1+\square} \\
 & & & \frac{EA}{\ell}\cdot\frac{\square}{1+\square} & -\frac{EA}{\ell}\cdot\frac{3C^L}{2(1+\square)} & EA\cdot\frac{C^L}{2(1+\square)} \\
 & \text{SYM.} & & & \frac{3EI}{\ell^3}\cdot\frac{1+4\square}{1+\square} & -\frac{3EI}{\ell^2}\cdot\frac{1+2\square}{1+\square} \\
 & & & & & \frac{3EI}{\ell}\cdot\frac{3+4\square}{3(1+\square)}
\end{bmatrix}$$

$$\overline{\mathbf{X}}_t^{(p)} = \overline{\mathbf{X}}_t^{L}$$

$$\begin{pmatrix}
\frac{\square/C^L}{1+\square}\cdot\frac{R'_i}{\ell} \\
\frac{1}{1+\square}\cdot\frac{3}{2\ell}R'_i \\
\frac{1}{1+\square}\cdot R'_i \\
-\frac{\square/C^L}{1+\square}\cdot\frac{R'_i}{\ell} \\
-\frac{1}{1+\square}\cdot\frac{3}{2\ell}R'_i \\
\frac{1}{1+\square}\cdot\frac{R'_i}{2}
\end{pmatrix}$$

where $R'_i=\mathrm{sign}(M_{zi})R_i$, $C^L=\mathrm{sign}(M_{zi})AZ_{pi}/\mathrm{sign}(F_{xi})A_{pi}\ell$, $\square=(EA\ell^2/4EI)(C^L)^2$

a) In case of failure at the left-hand end

$$\mathbf{k}_t^{(p)} = \mathbf{k}_t^{R}$$

$$\begin{bmatrix}
\frac{EA}{\ell}\cdot\frac{1}{1+\boxslash} & \frac{EA}{\ell}\cdot\frac{3C^R}{2(1+\boxslash)} & EA\cdot\frac{C^R}{2(1+\boxslash)} & -\frac{EA}{\ell}\cdot\frac{1}{1+\boxslash} & -\frac{EA}{\ell}\cdot\frac{3C^R}{2(1+\boxslash)} & EA\cdot\frac{C^R}{1+\boxslash} \\
 & \frac{3EI}{\ell^3}\cdot\frac{1+4\boxslash}{1+\boxslash} & \frac{3EI}{\ell^2}\cdot\frac{1+2\boxslash}{1+\boxslash} & -\frac{EA}{\ell}\cdot\frac{3C^R}{2(1+\boxslash)} & -\frac{3EI}{\ell^3}\cdot\frac{1+4\boxslash}{1+\boxslash} & \frac{6EI}{\ell^2}\cdot\frac{\boxslash}{1+\boxslash} \\
 & & \frac{3EI}{\ell}\cdot\frac{3+4\boxslash}{3(1+\boxslash)} & -EA\cdot\frac{C^R}{2(1+\boxslash)} & -\frac{3EI}{\ell^2}\cdot\frac{1+2\boxslash}{1+\boxslash} & \frac{2EI}{\ell}\cdot\frac{\boxslash}{1+\boxslash} \\
 & & & \frac{EA}{\ell}\cdot\frac{1}{1+\boxslash} & \frac{EA}{\ell}\cdot\frac{3C^R}{2(1+\boxslash)} & -EA\cdot\frac{C^R}{1+\boxslash} \\
 & \text{SYM.} & & & \frac{3EI}{\ell^3}\cdot\frac{1+4\boxslash}{1+\boxslash} & -\frac{6EI}{\ell^2}\cdot\frac{\boxslash}{1+\boxslash} \\
 & & & & & \frac{4EI}{\ell}\cdot\frac{\boxslash}{1+\boxslash}
\end{bmatrix}$$

$$\overline{\mathbf{X}}_t^{(p)} = \overline{\mathbf{X}}_t^{R}$$

$$\begin{pmatrix}
-\frac{\boxslash/C^R}{1+\boxslash}\cdot\frac{R'_j}{\ell} \\
\frac{1}{1+\boxslash}\cdot\frac{3}{2\ell}R'_j \\
\frac{1}{1+\boxslash}\cdot\frac{R'_j}{2} \\
\frac{\boxslash/C^R}{1+\boxslash}\cdot\frac{R'_j}{\ell} \\
-\frac{1}{1+\boxslash}\cdot\frac{3}{2\ell}R'_j \\
\frac{1}{1+\boxslash}\cdot R'_j
\end{pmatrix}$$

where $R'_j=\mathrm{sign}(M_{zi})R_j$, $C^R=\mathrm{sign}(M_{zj})AZ_{pj}/\mathrm{sign}(F_{xj})A_{pj}\ell$, $\boxslash=(EA\ell^2/4EI)(C^R)^2$

b) In case of failure at the right-hand end

$$\mathbf{k}_t^{(p)} = \mathbf{k}_t^{LR}$$

$$\begin{bmatrix}
\frac{EA}{\ell}\cdot\frac{3}{\boxed{}} & -\frac{EA}{\ell}\cdot\frac{3(C^L-C^R)}{\boxed{}} & -EA\cdot\frac{3C^L}{\boxed{}} & -\frac{EA}{\ell}\cdot\frac{3}{\boxed{}} & \frac{EA}{\ell}\cdot\frac{3(C^L-C^R)}{\boxed{}} & EA\cdot\frac{3C^R}{\boxed{}} \\
 & \frac{12EI}{\ell^3}\cdot\frac{\square+\boxslash-2\boxtimes}{\boxed{}} & \frac{12EI}{\ell^2}\cdot\frac{\square-\boxtimes}{\boxed{}} & \frac{EA}{\ell}\cdot\frac{3(C^L-C^R)}{\boxed{}} & -\frac{12EI}{\ell^3}\cdot\frac{\square+\boxslash-2\boxtimes}{\boxed{}} & \frac{12EI}{\ell^2}\cdot\frac{\boxslash-\boxtimes}{\boxed{}} \\
 & & \frac{12EI}{\ell}\cdot\frac{\square}{\boxed{}} & EA\cdot\frac{3C^L}{\boxed{}} & -\frac{12EI}{\ell^2}\cdot\frac{\square-\boxtimes}{\boxed{}} & -\frac{12EI}{\ell}\cdot\frac{\boxtimes}{\boxed{}} \\
 & & & \frac{EA}{\ell}\cdot\frac{3}{\boxed{}} & -\frac{EA}{\ell}\cdot\frac{3(C^L-C^R)}{\boxed{}} & -EA\cdot\frac{3C^R}{\boxed{}} \\
 & \text{SYM.} & & & \frac{12EI}{\ell^3}\cdot\frac{\square+\boxslash-2\boxtimes}{\boxed{}} & -\frac{12EI}{\ell^2}\cdot\frac{\boxslash-\boxtimes}{\boxed{}} \\
 & & & & & \frac{12EI}{\ell}\cdot\frac{\boxslash}{\boxed{}}
\end{bmatrix}$$

$$\overline{\mathbf{X}}_t^{(p)} = \overline{\mathbf{X}}_t^{LR}$$

$$\begin{pmatrix}
\frac{4\square+2\boxtimes}{\boxed{}\,\ell}\cdot\frac{R'_i}{C^L} - \frac{4\boxslash+2\boxtimes}{\boxed{}\,\ell}\cdot\frac{R'_j}{C^R} \\
\frac{3(1+2\boxslash+2\boxtimes)}{\boxed{}}\cdot\frac{R'_i}{\ell} + \frac{3(1+2\square+2\boxtimes)}{\boxed{}}\cdot\frac{R'_j}{\ell} \\
\frac{3+4\boxslash+2\boxtimes}{\boxed{}}\cdot R'_i + \frac{2(\square+2\boxtimes)}{\boxed{}}\cdot R'_j \\
-\frac{4\square+2\boxtimes}{\boxed{}\,\ell}\cdot\frac{R'_i}{C^L} + \frac{4\boxslash+2\boxtimes}{\boxed{}\,\ell}\cdot\frac{R'_j}{C^R} \\
-\frac{3(1+2\boxslash+2\boxtimes)}{\boxed{}}\cdot\frac{R'_i}{\ell} - \frac{3(1+2\square+2\boxtimes)}{\boxed{}}\cdot\frac{R'_j}{\ell} \\
\frac{2(\boxslash+2\boxtimes)}{\boxed{}}\cdot R'_i + \frac{(3+4\square+2\boxtimes)}{\boxed{}}\cdot R'_j
\end{pmatrix}$$

where $\boxtimes=(EA\ell^2/4EI)(C^LC^R)$, $\boxed{}=3+4\square+4\boxslash+4\boxtimes$

c) In case of Failure at the both ends

Fig. 3 Reduced element stiffness matrix $k_i^{(p)}$, and equivalent nodal force vector $\overline{X}_i^{(p)}$ for a plane frame structure.

of an element be serially numbered. Here, the failure criterion of the i-th element end is given by

$$Z_i = R_i - \boldsymbol{C}_i^T \boldsymbol{X}_t \leqq 0 \tag{10}$$

Structural failure of a frame structure is defined as occurrence of the plastic collapsing in the structure. A criterion for structural failure is given in the following manner. When the element ends r_1, r_2, $\cdots$, r_{p-1} have failed, stress analysis is performed once again and the element stiffness equation is obtained as

$$\boldsymbol{X}_t = \boldsymbol{k}_t^{(p)} \boldsymbol{\delta}_t + \overline{\boldsymbol{X}}_t^{(p)} \tag{11}$$

where $\boldsymbol{k}_t^{(p)}$: reduced element stiffness matrix
$\overline{\boldsymbol{X}}_t^{(p)}$: equivalent nodal force vector

After calculating the reduced element stiffness matrix for all the elements, they are assembled to have the total structure stiffness equation:

$$\boldsymbol{K}^{(p)} \boldsymbol{d} = \boldsymbol{L} + \boldsymbol{R}^{(p)} \tag{12}$$

where $\boldsymbol{d}$: total nodal displacement vector referred to the global coordinate system
$\boldsymbol{K}^{(p)} = \sum_{t=1}^{n} \boldsymbol{T}_t^T \boldsymbol{k}_t^{(p)} \boldsymbol{T}_t$: reduced total structure stiffness matrix
$\boldsymbol{T}_t$: transformation matrix
$\boldsymbol{L}$: vector of the external loads
$\boldsymbol{R}^{(p)} = -\sum_{t=1}^{n} \boldsymbol{T}_t^T \overline{\boldsymbol{X}}_t^{(p)}$: equivalent nodal force vector referred to the global coordinate system

Solving the above equation with respect to the nodal displacement vector yields

$$\boldsymbol{d} = [\boldsymbol{K}^{(p)}]^{-1} (\boldsymbol{L} + \boldsymbol{R}^{(p)}) \tag{13}$$

From Eq. (13), the nodal displacement vector $\boldsymbol{d}_t$ of the t-th element referred to the global coordinate system is given by

$$\boldsymbol{d}_t = [\boldsymbol{K}_t^{(p)}]^{-1} (\boldsymbol{L} + \boldsymbol{R}^{(p)}) \tag{14}$$

where the matrix $[\boldsymbol{K}_t^{(p)}]^{-1}$ indicates the matrix formed by extracting the rows corresponding to the vector $\boldsymbol{d}_t$ from the matrix $[\boldsymbol{K}^{(p)}]^{-1}$. As $\boldsymbol{\delta}_t$ is related to $\boldsymbol{d}_t$ through the transformation matrix $\boldsymbol{T}_t$: $\boldsymbol{\delta}_t = \boldsymbol{T}_t \boldsymbol{d}_t$, the nodal force vector $\boldsymbol{X}_t$ of the t-th member is given by

$$\boldsymbol{X}_t = \boldsymbol{b}_t^{(p)} (\boldsymbol{L} + \boldsymbol{R}^{(p)}) + \overline{\boldsymbol{X}}_t^{(p)} \tag{15}$$

where $\boldsymbol{b}_t^{(p)} = \boldsymbol{k}_t^{(p)} \boldsymbol{T}_t [\boldsymbol{K}_t^{(p)}]^{-1}$

After repeating the above processes, structural failure results when the element ends up to some specified number p_k, *e.g.*, element ends r_1, r_2, $\cdots$, and r_{p_k}, have failed. Occurrence of the plastic collapsing is determined by investigating the total structure stiffness matrix $[\boldsymbol{K}^{(p_k)}]$. For example, a criterion for structural failure is given by

$$|[K^{(p_k)}]|\leqq\varepsilon \tag{16}$$

where ε is a specified constant. Another criterion will be afforded by the magnitudes of the nodal displacements.

Now let us consider the expressions for the safety margins of the element ends. When the element ends r_1, r_2, $\cdots$, and r_{p-1} have failed, the safety margin of the surviving element end i (element number t) is given by

$$Z_i^{(p)}=R_i+C_i^T(b_i^{(p)}\sum_{k=1}^{n}T_k^T\overline{X}_k^{(p)}-\overline{X}_t^{(p)})-C_i^T b_i^{(p)}L \tag{17}$$

$$=R_i+\sum_{k=1}^{p-1}a_{ir_k}R_{r_k}-\sum_{j=1}^{ml}b_{ij}L_j \tag{18}$$

Note that Eq. (17) results by substitution of Eq. (15) into Eq. (10), and Eq. (18) does by resolution of the vectors into their components.

By using the safety margins, a criterion of structural failure is given by

$$Z_{r_p}^{(p)}\leqq 0 \qquad (p=1, 2, \cdots, p_k) \tag{19}$$

If there are any failed element ends r_p, which have their coefficients $a_{r_{p_k}r_p}$ equal to zero in the safety margin of the last yielded element end r_{p_k}, *i.e.*,

$$a_{r_{p_k}r_p}=0 \tag{20}$$

they are the redundant element ends which do not directly contribute to occurrence of the plastic collapsing.

In the searching process of a complete failure path, the value of λ_k ($k=i, j$) has an important physical meaning as described below. When λ_k ($k=i, j$) satisfies

$$\lambda_k\geqq 0 \tag{21}$$

in Eqs. (9), yielding of the element end continues. On the other hand, when

$$\lambda_k<0 \tag{22}$$

unloading has started. Consequently, in case of $\lambda_k<0$, it is neccessary to eliminate the element end k from the set of the failed element ends to form a complete failure path.

In summary, the plasticity condition of the element end under the combined loads has been approximated by a linear surface given by Eq. (1) regardless of a plane or space structure, and the safety margin of the element end has been expressed as a linear combination of the strengths of the element ends and the applied loads. Consequently, reliability analysis is greatly facilitated when the strengths and the loads are random variables.

4. Conclusions

The method was developed in this paper for generating the safety margins of the frame structures to perform their reliability analysis under the combined loading conditions. The plasticity condition of the element section was approximated by a linear surface and structural failure was determined by production of large nodal displacements due to the plastic collapsing. The matrix method was applied to have the expression of the safety margins and the failure criteria. The conclusions are summarized as follows:

1. The reduced stiffness matrix describing the elastic-plastic behaviour of the elements and the equivalent nodal forces after development of plastic hinges are derived by approximating the plasticity condition with a linear surface.
2. By using the matrix method, the safety margins of the element ends are expressed as linear combinations of the strengths of the elements and the applied loads.
3. The proposed method can be applied to generation of the safety margins both for plane and space frame structures under any loading conditions.

Acknowledgements

The authors would like to greatly appreciate the discussions by Prof. M. Yonezawa and Mr. M. Kishi during the course of this study. This work is financially supported by a Grant-in-Aid for Scientific Research which is provided by the Ministry of Education, Science and Culture of Japan.

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