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**Policy Mix for Pareto Optimal Redistribution
in the Presence of Tax Evasion**

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ABSTRACT: A purpose of this paper is to analyze the relationship between pareto optimal redistribution and tax evasion. The tax evasion of two types are simultaneously examined; the first type of the tax evasion is made by understating the initial income of the rich and the second type of the tax evasion is made by overstating the voluntary transfer by the rich to the poor for getting the deduction. Following main results have been obtained; (a) raising the penalty rate of the tax evasion will decrease not only the expected value of the evaded tax by the understatement of the initial income and by the overstatement of the voluntary transfer but also the understatement rate of the initial income and the overstatement rate of the voluntary transfer, however it will also decrease the voluntary transfer by the rich, (b) raising the tax rate will increase the amount of the transfer but will have no effect on the understatement rate of the initial income and on the overstatement rate of the voluntary transfer, however, it will increase the expected value of the evaded tax by the understatement of the initial

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income and by the overstatement of the voluntary transfer, (c) the higher the tax rate, the stronger the negative effect of raising the penalty rate of the tax evasion on the voluntary transfer by the rich to the poor.

Policy mix for decreasing not only the understatement rate of the initial income of the rich, the overstatement rate of the voluntary transfer by the rich to the poor for getting the deduction at the calculation of the tax but also the expected total tax evasion without affecting voluntary transfer will also be analyzed in this paper.

Keywords: Pareto Optimal Redistribution, Tax Evasion, Voluntary Transfer.

1. Introduction

A purpose of this paper is to analyze the relationship between pareto optimal redistribution (See Hochman and Rodgers (1969, 1974), and Musgrave (1970)) and tax evasion (See Allingham and Sandmo (1972), Laszlo (2004), Peacock and Show 1982), and Watanabe (1986, 1987, 1988, 1989, 2005, 2011)), as the relationship has not been examined.

In the following, it is assumed that a part of the voluntary transfer by the rich to the poor is deducted from the initial income of the rich, when the tax of the rich is calculated, and the income of the poor is not taxed.

In section 2, a simple but generalized pareto optimal redistribution model will be presented, taking the tax evasion into consideration. The tax evasion of two types are simultaneously examined; the first type of the tax evasion is made by understating the initial income of the rich and the second type of the tax evasion is made by overstating the voluntary transfer by the rich to the poor.

From the analysis of this paper following results have been obtained; (i) raising the penalty rate of the tax evasion will decrease not only the expected value of the evaded tax by the understatement of the initial income and by overstatement of the voluntary transfer but also the understatement rate of the initial income and the overstatement rate of the voluntary transfer, however it will also decrease the voluntary transfer by the rich, (ii) raising the tax rate will increase the amount of the transfer but will have no effect

on the understatement rate of the initial income and on the overstatement rate of the voluntary transfer, however, it will increase the expected value of the evaded tax by understatement of the initial income and by overstatement of the voluntary transfer, (iii) similarly, raising the degree of the deduction with respect to the voluntary transfer when the tax of the rich is calculated, will increase the amount of the transfer but will have no effect on the understatement rate of the initial income and the overstatement rate of the voluntary transfer, however, it will increase the expected value of the evaded tax, (iv) the higher the tax rate, the stronger the negative effect of raising the penalty rate of the tax evasion on the voluntary transfer by the rich to the poor, (v) the higher the penalty rate of the tax evasion, the weaker the positive effect of raising the degree of the deduction concerning the voluntary transfer at the time the tax is calculated on the voluntary transfer, (vi) the higher the deduction rate concerning the voluntary transfer at the calculation of the tax, the higher the positive effect of raising the tax rate on the voluntary transfer by the rich to the poor.

Policy mix for decreasing not only the understatement rate of the initial income of the rich, the overstatement rate of the voluntary transfer by the rich to the poor but also the expected total tax evasion without affecting voluntary transfer will also be analyzed in this paper.

In the last section concluding remarks will be given.

2. A Simple Model of Pareto Optimal Redistribution in the Presence of Tax Evasion

In the following, it is assumed that a part of the voluntary transfer, Y_T , by the rich to the poor is deducted from the initial income, Y_A° , of the rich, when the tax of the rich is calculated, and the income of the poor is not taxed.

Hence, the amount of the tax paid by the rich is equal to $t(Y_A^\circ - \lambda Y_T)$ and the after tax income becomes $Y_A^\circ - t(Y_A^\circ - \lambda Y_T) - Y_T$, where λ denotes the degree of the deduction.

On the other hand, the income of the poor becomes $Y_B^\circ + Y_T$, where Y_B° is the initial income of the poor.

The expected utility of the altruistic rich is denoted by the following (1).

$$\begin{aligned}
EU &= \alpha [Y_A^\circ - t \{(1 - \epsilon) Y_A^\circ - \lambda (1 - \delta) Y_T\} - Y_T] \\
&\quad + \beta \log(Y_B^\circ + Y_T) \\
&\quad - \epsilon (1 - \delta) \alpha F t \epsilon Y_A^\circ \\
&\quad - (1 - \epsilon) \delta \alpha F t \lambda \delta Y_T \\
&\quad - \epsilon \delta \alpha (\epsilon Y_A^\circ + \lambda \delta Y_T) F t,
\end{aligned} \tag{1}$$

where ϵ is the rate of the understatement of the income, δ is the rate of the overstatement of the transfer, and altruistic utility function is specified such that

$$u = \alpha Y_A + \beta \log Y_B, \quad \alpha > 0, \quad \beta > 0,$$

and

t is the tax rate, F is the penalty rate of the tax evasion.

Maximizing the expected utility with respect to Y_T , ϵ and δ yields the following first order conditions;

$$\begin{aligned}
\frac{\partial EU}{\partial Y_T} &= \alpha \left\{ t \lambda \left(1 + \frac{1}{4F} \right) - 1 \right\} + \frac{\beta}{Y_B^\circ + Y_T} \\
&= 0,
\end{aligned} \tag{2}$$

$$\begin{aligned}
\frac{\partial EU}{\partial \epsilon} &= \alpha t Y_A^\circ (1 - 2F\epsilon) \\
&= 0,
\end{aligned} \tag{3}$$

$$\begin{aligned}
\frac{\partial EU}{\partial \delta} &= \alpha t \lambda Y_T (1 - 2F\delta) \\
&= 0.
\end{aligned} \tag{4}$$

Second order conditions are satisfied.

$$\frac{\partial^2 EU}{\partial Y_T^2} = -\beta (Y_B^\circ + Y_T)^{-2} < 0, \tag{5}$$

$$\begin{vmatrix} \frac{\partial^2 \text{EU}}{\partial Y_T^2} & \frac{\partial^2 \text{EU}}{\partial Y_T \partial \epsilon} \\ \frac{\partial^2 \text{EU}}{\partial \epsilon \partial Y_T} & \frac{\partial^2 \text{EU}}{\partial \epsilon^2} \end{vmatrix} > 0, \quad (6)$$

where $\frac{\partial^2 \text{EU}}{\partial Y_T \partial \epsilon} = 0$, and $\frac{\partial^2 \text{EU}}{\partial \epsilon^2} = -2\alpha t Y_A^\circ F < 0$,

$$\begin{vmatrix} \frac{\partial^2 \text{EU}}{\partial Y_T^2} & \frac{\partial^2 \text{EU}}{\partial Y_T \partial \epsilon} & \frac{\partial^2 \text{EU}}{\partial Y_T \partial \delta} \\ \frac{\partial^2 \text{EU}}{\partial \epsilon \partial Y_T} & \frac{\partial^2 \text{EU}}{\partial \epsilon^2} & \frac{\partial^2 \text{EU}}{\partial \epsilon \partial \delta} \\ \frac{\partial^2 \text{EU}}{\partial \delta \partial Y_T} & \frac{\partial^2 \text{EU}}{\partial \delta \partial \epsilon} & \frac{\partial^2 \text{EU}}{\partial \delta^2} \end{vmatrix} < 0, \quad (7)$$

where $\frac{\partial^2 \text{EU}}{\partial Y_T \partial \delta} = 0$, $\frac{\partial^2 \text{EU}}{\partial \epsilon \partial \delta} = 0$, and $\frac{\partial^2 \text{EU}}{\partial \delta^2} = -2\alpha t \lambda Y_T F < 0$.

From the first order conditions (2), (3), and (4), the pareto optimal transfer or redistribution, Y_T^* , ϵ^* , and δ^* in this model where the tax evasion is taken into consideration, are obtained.

$$Y_T^* = \frac{\beta}{\alpha \left\{ 1 - t\lambda \left(1 + \frac{1}{4F} \right) \right\}} - Y_B^\circ, \quad (8)$$

$$\epsilon^* = \frac{1}{2F}, \quad (9)$$

$$\delta^* = \frac{1}{2F}. \quad (10)$$

Hence, the following results can straightforwardly be derived;

$$\frac{\partial Y_T^*}{\partial F} = -\frac{\beta}{4\alpha} \left\{ 1 - t\lambda \left(1 + \frac{1}{4F} \right) \right\}^{-2} t\lambda F^{-2} < 0, \quad (11)$$

$$\frac{\partial \epsilon^*}{\partial F} < 0, \quad (12)$$

$$\frac{\partial \delta^*}{\partial F} < 0. \quad (13)$$

Therefore, raising the penalty rate of the tax evasion will decrease the understatement rate of the income, the overstatement rate of the transfer and the amount of the voluntary transfer by the rich to the poor.

In the same way, additional results can also be obtained.

$$\frac{\partial Y_T^*}{\partial \lambda} = \frac{\beta}{\alpha} \left\{ 1 - t\lambda \left(1 + \frac{1}{4F} \right) \right\}^{-2} t \left(1 + \frac{1}{4F} \right) > 0, \quad (14)$$

$$\frac{\partial \epsilon^*}{\partial \lambda} = 0, \quad (15)$$

$$\frac{\partial \delta^*}{\partial \lambda} = 0, \quad (16)$$

$$\frac{\partial Y_T^*}{\partial t} = \frac{\beta}{\alpha} \left\{ 1 - t\lambda \left(1 + \frac{1}{4F} \right) \right\}^{-2} \lambda \left(1 + \frac{1}{4F} \right) > 0, \quad (17)$$

then from (14) and (17)

$$\frac{\partial Y_T^*}{\partial \lambda} \underset{\cong}{\cong} \frac{\partial Y_T^*}{\partial t} > 0, \text{ according as } t \underset{\cong}{\cong} \lambda.$$

$$\frac{\partial \epsilon^*}{\partial t} = 0, \quad (18)$$

$$\frac{\partial \delta^*}{\partial t} = 0. \quad (19)$$

Similarly, following results can also be derived straightforwardly.

$$\begin{aligned}\frac{\partial^2 Y_T^*}{\partial t \partial \lambda} &= 2 \frac{\beta}{\alpha} t \lambda \left(1 + \frac{1}{4F}\right)^2 \left\{1 - t \lambda \left(1 + \frac{1}{4F}\right)\right\}^{-3} \\ &\quad + \frac{\beta}{\alpha} \left\{1 - t \lambda \left(1 + \frac{1}{4F}\right)\right\}^{-2} \left(1 + \frac{1}{4F}\right) \\ &> 0,\end{aligned}\tag{20}$$

where $\left\{1 - t \lambda \left(1 + \frac{1}{4F}\right)\right\} > 0$, as $Y_T^* > 0$ from (8) is assumed.

$$\begin{aligned}\frac{\partial^2 Y_T^*}{\partial F \partial t} &= \frac{-\beta}{2\alpha} \left\{1 - t \lambda \left(1 + \frac{1}{4F}\right)\right\}^{-3} \lambda^2 t \left(1 + \frac{1}{4F}\right) F^{-2} \\ &\quad - \frac{\beta}{4\alpha} \left\{1 - t \lambda \left(1 + \frac{1}{4F}\right)\right\}^{-2} \lambda F^{-2} < 0,\end{aligned}\tag{21}$$

$$\begin{aligned}\frac{\partial^2 Y_T^*}{\partial \lambda \partial F} &= \frac{-\beta}{2\alpha} \left\{1 - t \lambda \left(1 + \frac{1}{4F}\right)\right\}^{-3} t^2 \lambda \left(1 + \frac{1}{4F}\right) F^{-2} \\ &\quad - \frac{\beta}{4\alpha} \left\{1 - t \lambda \left(1 + \frac{1}{4F}\right)\right\}^{-2} t F^{-2} \\ &< 0.\end{aligned}\tag{22}$$

Hence, an increase in λ will raise the positive effect of increasing the tax rate on the voluntary transfer by the rich to the poor from (17) and (20).

And the higher the tax rate, the stronger the negative effect of increasing the penalty rate of the tax evasion on the voluntary transfer from (11) and (21). From (14) and (22), the higher the penalty rate of the tax evasion, the weaker the positive effect of raising the degree of the deduction concerning the voluntary transfer at the time the tax is calculated on the voluntary transfer.

On the other hand, the expected value of the tax evasion, ETV , is denoted by

$$\begin{aligned}ETV^* &= t \epsilon^* Y_A^\circ + t \lambda \delta^* Y_T^* \\ &= t \frac{1}{2F} Y_A^\circ + t \lambda \frac{1}{2F} \left[\frac{\beta}{\alpha \left\{1 - t \lambda \left(1 + \frac{1}{4F}\right)\right\}} - Y_B^\circ \right].\end{aligned}\tag{23}$$

Following result can be obtained.

$$\frac{\partial ETV^*}{\partial t} > 0. \quad (24)$$

Hence, from (18), (19), and (24) raising the tax rate will have no effect on ϵ^* and δ^* , but increase the expected value of the tax evasion.

In the same way, the following results can be derived.

$$\frac{\partial ETV^*}{\partial F} < 0, \quad (25)$$

$$\frac{\partial ETV^*}{\partial \lambda} > 0. \quad (26)$$

Therefore, an increase in the penalty rate of the tax evasion or a decrease in the deduction rate of the voluntary transfer at the time tax is calculated, will reduce the expected value of the tax evasion.

Next, the policy mix for decreasing not only the understatement rate of the initial income of the rich, the overstatement rate of the voluntary transfer by the rich to the poor for getting the deduction at the calculation of the tax but also the expected total tax evasion without affecting voluntary transfer by the rich to the poor will be examined.

From (11), (12), (17), (18), (23), (24), and (25), it is straightforwardly shown that raising both the penalty rate of the tax evasion and the tax rate subject to the following condition (27) will decrease not only the understatement rate of the initial income of the rich, the overstatement rate of the voluntary transfer by the rich to the poor for getting the deduction at the calculation of the tax but also the expected total tax evasion without affecting voluntary transfer by the rich to the poor.

$$\eta = \frac{1}{4F + 1}, \quad (27)$$

$$\text{where } \eta \equiv \frac{\frac{dt}{t}}{\frac{dF}{F}}.$$

If, for example, $F = \frac{9}{4}$, then $\eta = 0.1$.

Therefore, raising F one percent with raising t 0.1 percent will be the policy mix for them.

3. Concluding Remarks

A purpose of this paper is to analyze the relationship between pareto optimal redistribution and tax evasion. The tax evasion of two types are simultaneously examined; the first type of the tax evasion is made by understating the initial income of the rich and the second type of the tax evasion is made by overstating the voluntary transfer by the rich to the poor. Following main results have been obtained; (a) raising the penalty rate of the tax evasion will decrease not only the expected value of the evaded tax by the understatement of the initial income and by the overstatement of the voluntary transfer but also the understatement rate of the initial income and the overstatement rate of the voluntary transfer, however it also decrease the voluntary transfer by the rich, (b) raising the tax rate will increase the amount of the transfer but will have no effect on the understatement rate of the initial income and on the overstatement rate of the voluntary transfer, however, it will increase the expected value of the evaded tax by understatement of the initial income and by overstatement of the voluntary transfer, (c) the higher the tax rate, the stronger the negative effect of raising the penalty rate of the tax evasion on the voluntary transfer by the rich to the poor.

Policy mix for decreasing not only the understatement rate of the initial income of the rich, the overstatement rate of the voluntary transfer by the rich to the poor for getting the deduction at the calculation of the tax but also the expected total tax evasion without affecting voluntary transfer is also analyzed in this paper. Raising both the penalty rate of the tax evasion and the tax rate under the condition examined above will decrease not only the understatement rate of the initial income of the rich, the overstatement rate of the voluntary transfer by the rich to the poor for getting the deduction at the calculation

of the tax but also the expected total tax evasion without affecting voluntary transfer by the rich to the poor.

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