

# A method of Transient Control Using Braking Resistors

著者	Kawabata Hiroaki, Kido Masao
引用	Bulletin of University of Osaka Prefecture. Series A, Engineering and natural sciences. 1977, 26(1), p.39-46
URL	<a href="http://doi.org/10.24729/00008696">http://doi.org/10.24729/00008696</a>

# A Method of Transient Control Using Braking Resistors

Hiroaki KAWABATA\* and Masao KIDO\*

(Received June 4, 1977)

There are several methods to remove the oscillatory transients due to a fault in a power system. Interest in stabilizing the transient oscillation is not new, but recently the results of modern control theory have been used for this purpose. In general, the system equations of the generators in transient period are nonlinear, and the mechanism of the motion in a multi-machine system is complicated.

This paper describes the procedure in applying the linear optimal control theory to a multi-machine power system's transient control using braking resistors.

## 1. Introduction

Depending on increases of electric power demand, the transmission power must be also increased, and the transient control scheme becomes more important from now on, in order to supply the power stably. For improvement of transient stability, it is of vital importance to know the absorption of excess kinetic energy by the braking resistors, the change of line reactances by insertion of the series capacitors, generator dropping, load shedding and so forth, so that proper design and rating may be selected for application to any given systems. In a one-machine infinite-bus system, the stability has been governed by the first swing of motion. But, in an actual power system it is observed to step out by successive swings. Consequently, the system stability must be considered in multi-machine system.

As to the control scheme, some trials of an optimal control to the transient control are investigated, but the problems lie in the nonlinearity of the system equation. Therefore, the most model system treated there is constrained to a one-machine infinite-bus system. Not only in one-machine infinite-bus system, but also in multi-machine system, it is necessary to solve the two-point boundary value problem that results, and it becomes very difficult depending on any increase of the dimensions. Moreover, the optimal controls obtained are open loop ones, and it is not of practical use. Only when the system is linear and the performance index is an integral of quadratic functions of the state variables and the control variables is there hope of obtaining solutions analytically. The optimal control is derived in the form of closed loop to be useful in many fields. As an approach to the system control of transient operations in a multi-machine system, the results are used as the suboptimal control to the original nonlinear system linearized the nonlinear power

---

\* Department of Electrical Engineering, College of Engineering.

equations and constructed the optimal controller to the linearized system. These procedures using the braking resistors are described in the next section.

## 2. Linearization and Optimal Control

In the case of power system with  $n$  generators, the equations of the  $i$ -th generator is expressed as follows:

$$M_i \frac{d^2 \delta_i}{dt^2} = P_{m_i} - P_{u_i}(\boldsymbol{\delta}, \mathbf{u}), \quad i=1, 2, \dots, n, \quad (1)$$

$$P_{u_i}(\boldsymbol{\delta}, \mathbf{u}) = G_{ii}(\mathbf{u}) E_i^2 + \sum_{j \neq i}^n \rho_{ij}(\mathbf{u}) \sin \{ \delta_i - \delta_j - \phi_{ij}(\mathbf{u}) \}, \quad (2)$$

where  $\delta_i$  = phase angle of the  $i$ -th generator,  
 $M_i$  = constant of inertia of the  $i$ -th generator,  
 $P_{m_i}$  = mechanical input of the  $i$ -th generator,  
 $E_i$  = voltage back of transient reactance,  
 $Y_{ij}$  = transfer admittance between the  $i$ -th and the  $j$ -th generators,  
 $\rho_{ij}(\mathbf{u}) = |Y_{ij}(\mathbf{u})| E_i E_j$ : coefficient of synchronizing power between the  $i$ -th and the  $j$ -th generators,  
 $\mathbf{u} = (u_1, u_2, \dots, u_r)$ : control vector, that is, a set of braking resistors,  
 $\phi_{ij}(\mathbf{u}) = \angle Y_{ij}(\mathbf{u})$ ,  
 $\boldsymbol{\delta} = (\delta_1, \delta_2, \dots, \delta_n)$ .

In order to simplify the equation, Eq. (1) may be rewritten as follows:

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}, \mathbf{u}), \quad (3)$$

where  $\mathbf{z} = (z_1, z_2, \dots, z_{2n}) = (\delta_1, \dot{\delta}_1, \delta_2, \dots, \delta_n, \dot{\delta}_n)$ ,  
 $\mathbf{f} = (f_1, f_2, \dots, f_{2n})$ ,  
 $f_{2i} = \frac{1}{M_i} \{ P_{m_i} - P_{u_i}(\mathbf{z}, \mathbf{u}) \}$ ,  $f_{2i-1} = z_{2i}$ .

By linearizing Eq. (3) around the stable equilibrium point, the following equation is obtained:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad \left( \mathbf{A} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{z}} \right|_{\mathbf{z}=\mathbf{z}_0}, \quad \mathbf{B} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right|_{\mathbf{u}=\mathbf{u}_0} \right), \quad (4)$$

$$\mathbf{x} = \mathbf{z} - \mathbf{z}_0. \quad (5)$$

Eq. (4) gives a considerable approximation of Eq. (3) within the comparatively wide range of

$$-\frac{\pi}{2} \leq \delta_i - \delta_j - \phi_{ij} \leq \frac{\pi}{2},$$

because Eq. (3), that is, Eq. (1) is made of the sinusoidal functions that gives a good approximation within the above range.

Next, to derive the optimal regulator, the following performance index is set:

$$J = \int_0^{\infty} (x' Q x + u' R u) dt. \quad (6)$$

As well-known, the optimal control to minimize the performance index (6) is given by

$$u = -R^{-1} B' K x, \quad (7)$$

where matrix  $K$  is the solution of the following Riccati-equation:

$$A' K + K A + Q - K B R^{-1} B' K = 0. \quad (8)$$

Based on these assumptions and considerations, it is possible to estimate the effectiveness of the suboptimal control on the original nonlinear system. The effectiveness of the suboptimal control in a three-machine model system will be shown in Section 4 and will be discussed at that time.

### 3. Braking Resistors

The braking resistors are located near a generating plant in each sending terminal and are connected in shunt with the three-phase bus through a suitable switch that is normally open. The switch is closed as soon as the fault has been cleared. The resistors have a control function made by absorbing the excess kinetic energy due to the fault. The conventional brake application should be used with energy absorption equal to or slightly greater than the excess kinetic energy. In a multi-machine system two or more successive applications are required on successive swings, and it is difficult to remove the oscillatory transients completely because of the interaction between the generators. But, by way of the optimal control, it is possible to absorb the excess kinetic energy in the system systematically. In order to apply the optimal regulator with quadratic cost function, it should be assumed that the capacity of resistors can be changed continuously or discretely. The threshold value will be set to take the actual application into consideration. All the energy absorbed raises the temperature of the resistors, and the brakes are usually given by a short-time rating.

### 4. Three-machine System

In this section, the procedures considered in previous sections are applied to the three-machine system shown in Fig. 1. The bus of each generating-station is equipped with braking resistors. Line resistances are neglected and line reactances are given in per unit on 100 MVA base. Data on the generators and on the initial generating-station outputs and bus voltages are given in Table 1.

#### 4.1 Determination of Matrices A and B

It is not possible to determine the values of A and B matrices analytically. To

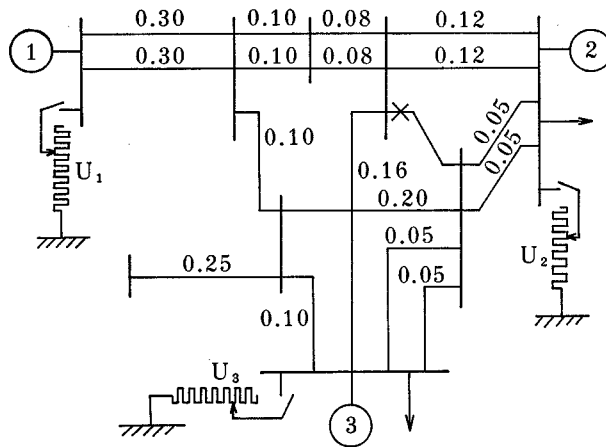


Fig. 1. Three-machine model system.

Table 1. Data for model system

Station	Transient Reactance (p.u.)	M (sec <sup>2</sup> /rad)	Initial Station Output (p.u.)	Load (p.u.)	Initial Bus Voltage (p.u.)
1	0.333	0.0167	0.8	0.0	1.05
2	0.070	0.1114	2.3	2.0	1.00
3	0.180	0.0424	0.9	2.0	1.00

estimate these values, numerical computations are used. In this model system, by setting  $\Delta\delta=0.1745$  (rad) (about  $10^\circ$ ),  $u_i=0.05$  (p.u.), the A and B matrices become the following:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -103.89 & 0 & 72.00 & 0 & 32.08 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 12.42 & 0 & -37.71 & 0 & 27.93 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 10.94 & 0 & 68.83 & 0 & -87.93 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ -31.36 & -4.72 & -5.47 \\ 0 & 0 & 0 \\ -3.25 & -5.77 & -4.40 \\ 0 & 0 & 0 \\ -4.62 & -5.21 & -7.95 \end{pmatrix}.$$

### 4.2 Suboptimal Control Using Braking Resistors

The swing curves are plotted in Fig. 2, assuming a three-phase short circuit to occur at point X in Fig. 1, and to be cleared in 0.11 second. The generators iterate the relative oscillation, and depart from the reference axis.

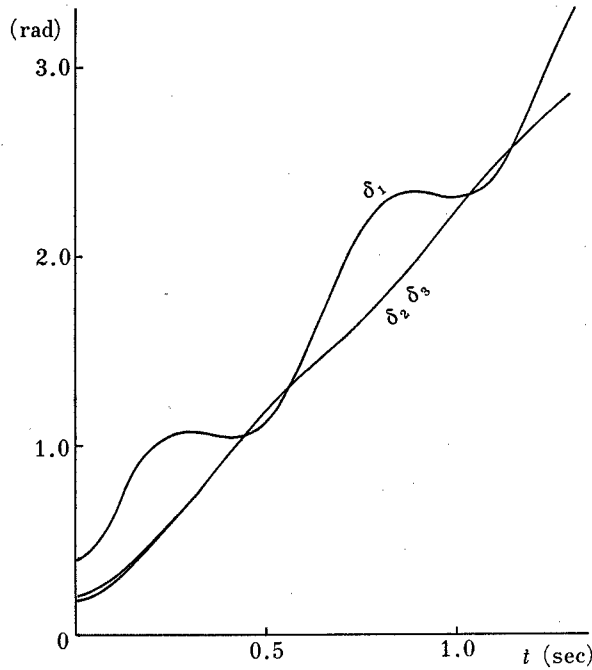


Fig. 2. The swing curves after the fault is cleared.

To apply the suboptimal control, as an example, weighting matrices Q and R are given as follows:

$$Q = \begin{pmatrix} 60 & & & \\ & 1 & & 0 \\ & & 60 & \\ & & & 1 \\ 0 & & & & 60 \\ & & & & & 1 \end{pmatrix},$$

$$R = r \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 200 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Then, the solution of Eq. (8), K matrix, is given by

$$K = \begin{pmatrix} 50.64 & 0.66 & -10.98 & 2.88 & -22.64 & 0.36 \\ 0.66 & 0.43 & 0.004 & 0.15 & 0.84 & -0.15 \\ -10.98 & 0.004 & 244.76 & 3.36 & -214.04 & 1.42 \\ 2.88 & 0.15 & 3.36 & 4.23 & 2.60 & -1.22 \\ -22.64 & 0.84 & -214.04 & 2.60 & 286.46 & 1.36 \\ 0.36 & -0.15 & 1.42 & -1.22 & 1.36 & 2.71 \end{pmatrix}$$

The characteristic curves in this  $K$  matrix are shown in Fig. 3. The transient oscillations are suppressed by use of the suboptimal control, but there are problems to be noted. The initial condition of controls is, in a simplified form, 0.3 p.u. as the threshold value, since for the range of resistance covered in this investigation the initial values of controls were usually large, and the effects of the negative control of the system are not included, though the error in neglecting this factor, in general, increases with the magnitude of the negative value. The characteristic curves with these sophisticated controls are shown in Figs. 4 and 5.

From the standpoint of stability, methods of the restricted controls will be satisfactory, for a study of the system in question, based on the considerations treated above, indicates that instability will not result. Moreover, typical treatments for a variety of parameter  $r$  are illustrated in Fig. 6.

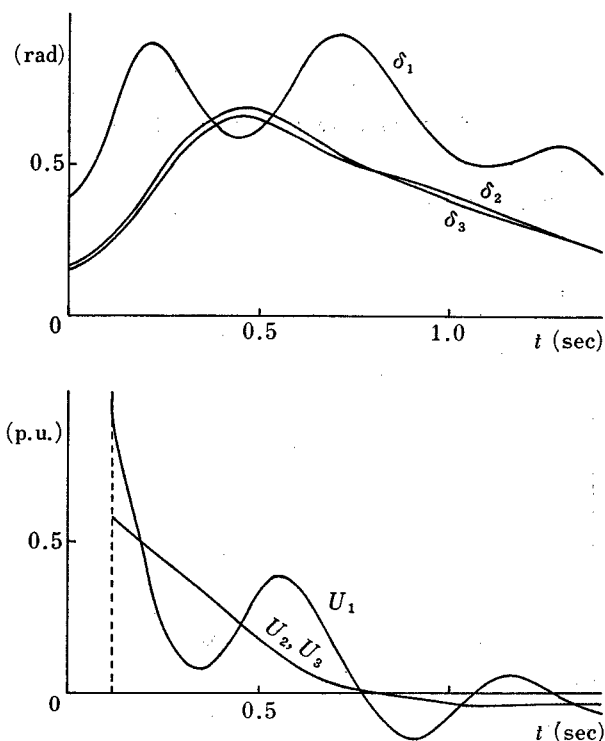


Fig. 3. The suboptimal controls and their trajectories.

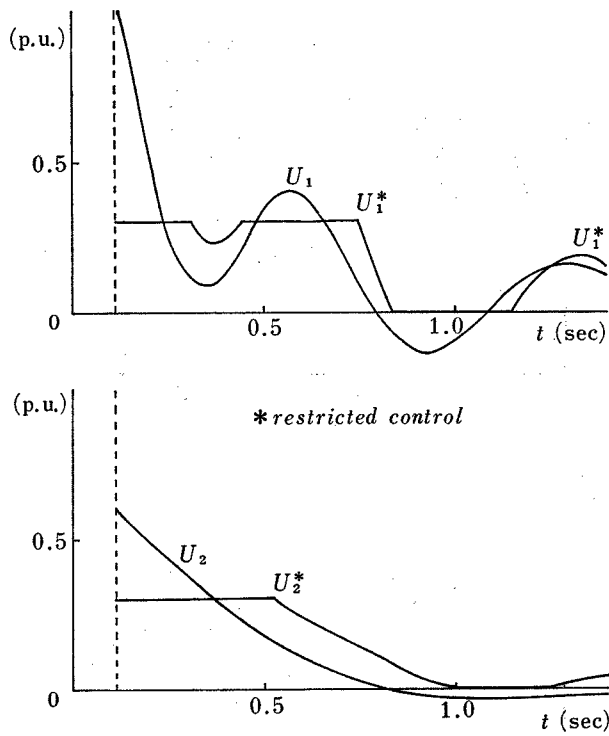


Fig. 4. The restricted controls.

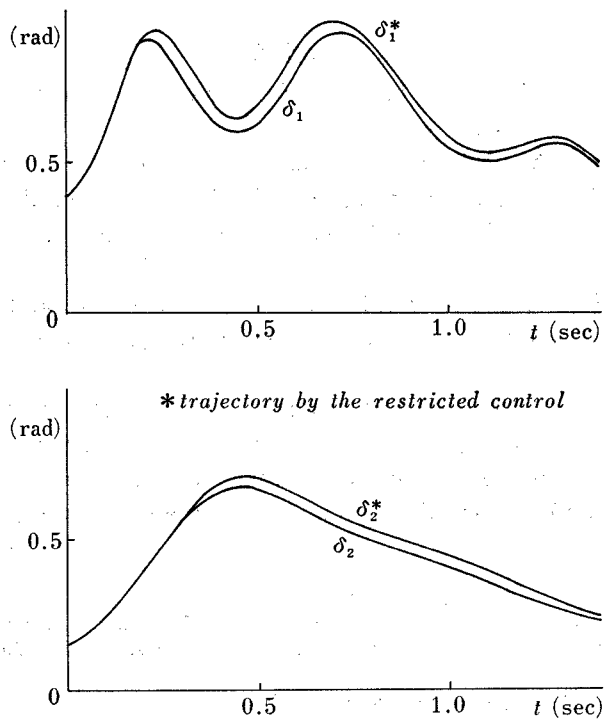


Fig. 5. Trajectories by the restricted control.



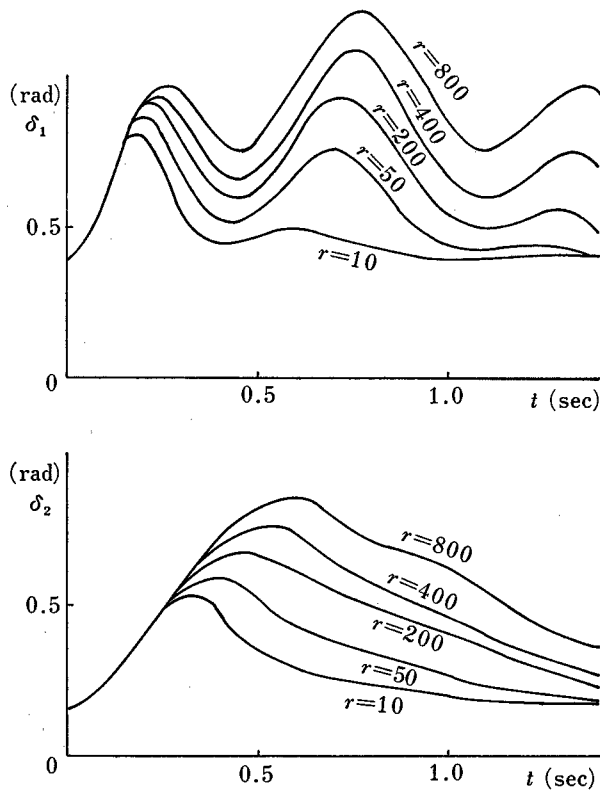


Fig. 6. Variation of trajectories by the parameter  $r$ .

### 5. Conclusion

The principal conclusions to be drawn from the analysis in this paper based on the optimal control theory are:

1) The results obtained cannot always be generalized to make them applicable to all systems. Certain general principles and effects have been discussed in order to show the trends in stability, but it may be advisable to get more detailed information about the actual system.

2) It must be emphasized that the methods of suboptimal control such as have been discussed here are really applicable to the actual system from the view point of system stability.

3) Though the control scheme using the braking resistors has been investigated, the same scheme may be also applied by using other devices.

4) It is still desirable to devise the method for the large oscillations over the linearized range, while the stability is improved in the considerably wide range by this method.

### References

- 1) E. W. Kimbark, *Power System Stability*, Vol 1, John Wiley & Sons, New York (1948).
- 2) H. Kwakernaak and R. Sivan, *Linear Optimal Control Systems*, John Wiley & Sons, New York (1972).
- 3) E. W. Kimbark, *IEEE Trans. PAS-88*, 773 (1969).
- 4) M. Kido, H. Kawabata, Y. Inagaki, Y. Shirao, A. Nakata, Lecture G4-29 in annual Meeting of Kansai Branch of I.E.E. of Japan (1974).