Optimization of Ship Fleet-Size

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A basic concept is proposed in this paper for determining the optimum fleet-size to meet a transport demand in an arbitrary route with one port loading and one port unloading. The quality of a fleet is considered to be judged based on the transport costs, which consist of link costs and node costs. Mathematical models relating the transport costs to the fleet-size, i. e., the size and service speed of a ship, the number of ships and their kinds, are developed for the case of crude oil carriers. The optimization problem is set up to determine the optimum fleet-size minimizing the transport costs, considering the technological and geometrical restrictions. For the solution of the problem, the concept of dynamic programming and nonlinear programming techniques are applied, and a versatile software program is developed. The effects of the transport-ation system's factors, such as the total transport demand, the draught limits, the tolls, the storage costs, etc., are discussed concerning the resulting optimum fleet-size.

1. Introduction

It is important for both developing and developed maritime nations to hold an optimum mercantile marine. Mercantile marine will contribute also to improving the international trade and payments and to fostering the shipbuilding industry and its allied industries. Thus, the optimum selection of the fleet-size for a specific purpose constitutes a key decision making particularly in capital investment, considering the effective utilization of the limited resources. For this purpose, fully taken into account must be the technological and economic aspects of the fleet. However, a macroscopic characterization of the fleet containing the essential factors of the fleet suffices to estimate the amount of the capital investment. Systems engineering techniques may provide us the powerful tools for accomplishing the above mentioned purpose. That is, analysis and modelling of the transportation system and optimization of the fleet may successfully be done by using the techniques developed in the field of systems engineering.

Studies so far made are mainly concerned with the economy of an individual ship design[1],[2],[3], and little has been done on the selection of the optimum fleet from the transportation system's point of view[4],[5].

A basic concept is proposed in this paper for determining an optimum fleet-size to meet a transport demand in an arbitrary route with one port loading and one port unloading. The quality of a fleet-size is considered to be judged based on the transport

costs, which consist of link costs and node costs. Mathematical models relating the transport costs to a fleet-size, i. e., the size and service speed of a ship, the number of ships and their kinds, are developed for the case of crude oil carriers. The optimization problem is set up to determine the optimum fleet-size minimizing the transport costs, considering the technological and geometrical restrictions concerned. The transport demand is assumed to be given for the route considered. For solving the problem, the concept of dynamic programming and nonlinear programming techniques are applied, and a versatile software program is developed for determining the optimum fleet-size. The effects of various factors, such as the transport demand, the draught limits, tolls, etc., on the optimum fleet-size are discussed.

2. Statement of Problem

2.1 Basic assumptions

Following assumptions are made of the models.

(1) Crude oil carriers are considered. Thus the transport demand is crude oil cargo.

(2) Transport between two ports are considered, and the transport flow is one sided, i. e., one port loading and one port unloading.

(3) Ships of identical age are considered.

(4) Ship arrivals are regular and no queues are considered at both ports.

2.2 Control variables

Control variables for determining the optimum fleet-size are the types of ships and the number of them. Types of ships are assumed to be represented by their dead weight and full load service speed, and the fleet is defined as a set of ships.

2.3 Optimality criterion

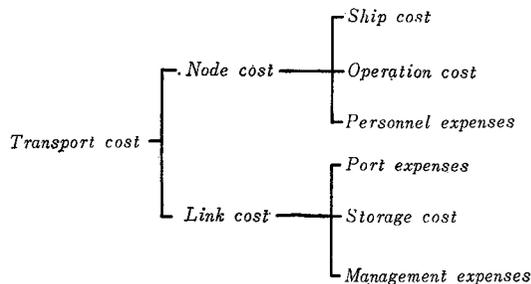As the criterion for the optimality of the fleet, the transport costs are considered.

Fig. 1 Transport costs

The transport costs are generally categorized as shown in Fig. 1. Of these costs, the costs depending on the control variables i. e., dead weight, service speed and the number of ships are taken into consideration for the determination of the optimum fleet-size. The detail of the models will be given in Section 4.

2.4 Technological and geometrical restrictions

Concerning the candidate types of ships to be selected, the following assumptions are made in consideration of the technological restrictions and the convenience of the treatment.

(1) The maximum and minimum dead weights of the member ships constituting the fleet are specified.

(2) The maximum and minimum service speeds of the ships are specified.

Further, considering the draught limits of the route, the maximum sizes of ships are taken into account for both full and ballast load conditions, corresponding to the conditions of the routes.

2.5 Problem

Given the transport demand between two ports, the following problem is considered. PROBLEM" Find the optimum fleet size, i. e., the payload and the service speed of a ship, the number of ships and their kinds, to minimize the transport costs, considering the technological and geometrical restrictions concerned."

3. Mathematical Models of Transport Costs

3.1 Link costs

Link costs which can be expressed in terms of the control variables and differ from fleet to fleet are considered. These are listed below:

(1) Ship costs

i) depreciation expenses ii) equipment fund interests iii) repair expenses

iv) insurances v) sundries

(2) Operation costs

i) fuel and oil expenses ii) tolls

(3) Personnel costs

Complement and reserves expenses

Mathematical models and cost data related to the calculation of the above quantities are given in the following sections.

3.1.1 Maximum operating power, fuel consumption and the number of crews.

The mathematical models of the maximum operating power (PSN), fuel consumption (FOC) and the number of crews (CRN for complement and CN with reserves) are built by using the data of 100 Japanese oil carriers[6], where the least square method is applied for the model building. The mathematical models and their multiple correlation coefficients are given in the following[7].

$$PSN=0.12553\times10^{-2}\ DW^{0.7}\ V^3+4325.913\ (HP),\ R=0.982 \tag{3-1}$$

$$FOC=0.447849\times10^{-2}\ PSN+13.692\ (ton/day),\ R=0.992\ \text{(turbine engine)} \tag{3-2}$$

$$=0.362478\times10^{-2}\ PSN+2.32131\ (ton/day),\ R=0.998\ \text{(diesel engine)} \tag{3-3}$$

$$CRN=1.63719\times DW^{0.2}+13.34658\ (persons) \tag{3-4}$$

$$CN=0.63380\times DW^{0.3}+14.07222\ (persons) \tag{3-5}$$

where DW and V are expressed in *dwt* and *kt*.

3.1.2 Ship construction costs

The mathematical models of the ship construction costs per tonnage (CC) and the turbine engine cost per HP (CME) are estimated by using the cost data in the latter half period of 1971[8]. The results are given as follows.

$$CC = 5.9 \times 10^2 \, DW^{-0.5} + 1.86 \quad (10^4 \, yen/dwt) \tag{3-6}$$

$$CME = 0.13651[(PSN/10^4)^2 - 9.0 \, (PSN/10^4) + 50.5] \quad (10^4 \, yen/HP)$$
$$\text{(for } PSN < 45000 HP)$$
$$= 4.129125 \tag{3-7}$$
$$\text{(for } PSN \geqq 45000 HP)$$

The construction costs of the ship with arbitrary dead weight and service speed are estimated based on the following assumptions:

(1) Main engines are of turbine type.

(2) The full load service speed is 16 kt for the standard type ships. The construction costs of the ships are given by Eq. (3-6).

(3) For ships with the service speed other than 16 kt, the variation in the construction costs is equal to that in the costs of the main engine.

Using the above assumptions, the constrution cost of the ship with dead weight DW dwt and service speed V kt is calculated by the formula:

$$CCN = CCS + (CEN - CES) \quad (10^4 yen/dwt) \tag{3-8}$$

where CCN = the construction cost of the ship with DW dwt and V kt

CCS = the construction cost of the standard type ship with DW dwt and V = 16 kt, calculated from Eq. (3-6)

CEN = the main engine cost of the ship with DW dwt and V kt, calculated from Eqs. (3-1) and (3-7)

CES = the main engine cost of the standard type ship with DW dwt and V = 16 kt, calculated from Eqs. (3-1) and (3-7).

For the illustration, the constrution costs are calculated and shown in Fig. 2 for the cases of V = 14, 16, 18 kts.
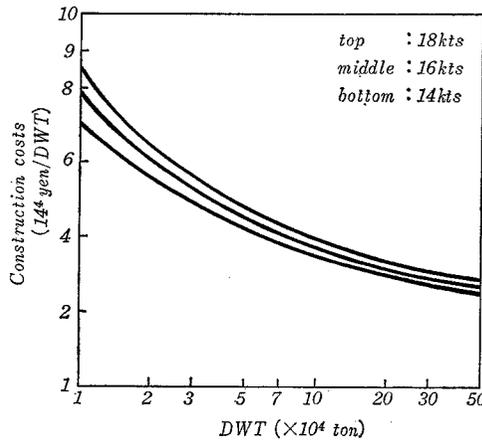

Fig. 2 Ship construction costs

3.1.3 Other cost data

The tolls passing the short cut route (see Section 4.2) are assumed to be

$$CCNL = C_\iota \times DW \quad (yen/passage) \tag{3-9}$$

where C_ι is a given constant.

Uniform annual returns are adopted as the method of depreciation. Cost data for the calculation of the link costs are given in Table 1. The interest rate of the

Table 1 Cost data

Items	Specific	Remark
Lay days	at port of shipment : 1.3 days at port of discharge : 2.0 days	each includes 0.5 spare day
Rate of operation	94%	
Sea speed at ballast condition	1.07 × (sea speed at full load condition)	$\alpha = 1.07$
Depreciation expenses	uniform annual returns life of service : 10 years residual value : 10% of original value	
Crew expenses	annual expenses per person : 3,000,000yen/man/year rising rate : 12% per year	
Repair expenses	10 years total sum/tonnage price$=0.1+0.004 \times DW \times 10^{-4}$	
Hull insurance	insurable value : tonnage price insurance rate : 1%	
Interest rate of equipment fund	See Table 2	based on the 30th Keikaku-zōsen
Sundries	200 yen/dwt	
Fuel expenses	cost of heavy oil : 6,000 yen/ton fuel consumption at port : 60% of "at voyage" fueling base : port of shipment	
Weights of fuel oil etc. (at departure)	fuel oil : (days at voyage+0.6days at port)·FOC fresh water : 10kg/person/day crew & their effects : 120kg/person provisions : 2.5kg/person/day un-statutory spares : $PSN \times 0.005$ ton	

Table 2 Interest rate of equipment fund

	Percentage	Annual interest	Repayment
Treasury investment	58% of tonnage price	6.5%	term of deferment : 2 years term of redemption : 8 years
Loan capital	22% of tonnage price	6.5%	term of redemption : 10 years
Fund on hand	20% of tonnage price	7.0%	term of redemption : 19 years
Fund on hand (working capital)	3% of tonnage price	7.0%	term of redemption : 10 years

equipment funds is based on the 30th Keikakuzosen (Japanese Government Ship Building Program) rate, and listed in Table 2.

3.2 Mathematical models of node costs

Only the port expenses and the storage costs are considered among the node costs, because it is very difficult to relate the management expenses to the control variables and these are presumed not to depend directly on the control variables.

3.2.1 Port expenses

The port expenses are generally composed of port dues, quarantine fee, customs charges, pilotage, light dues, towage, line handling charges, agency fees and sundries. The costs other than port dues, pilotage and light dues are almost constant regardless of the ship size in many ports. In this paper, a simple mathematical model:

$$PE = 25.0 \times DW \qquad (yen) \qquad (3\text{-}10)$$

is used. The formula (3-10)is compared with the actual data inFig. 3.
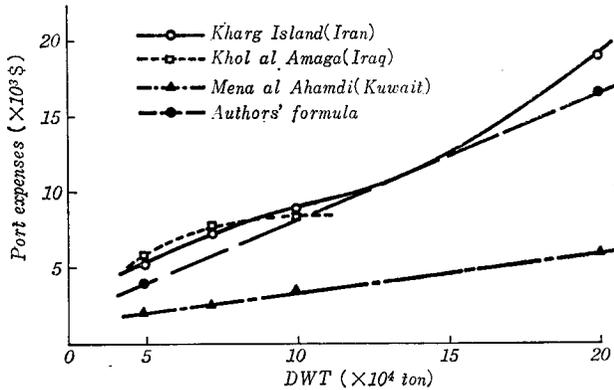

Fig. 3 Port expenses

3.2.2 Storage costs

Storage costs are dependent on the construction costs of the storage systems and their maintenance and administration expenses. Here, a simple mathematical model for the storage costs is presented. Assume that the annual total transport demand is TDT ton, and it is carried by the fleet of the identical ships. Denoting the transport capacity per trip in tonnage and the average time between the ship arrivals in days by ST and T, the variation of the inventory in the mean becomes as shown in Fig. 4.

Denoting the storage cost per day per tonnage by S_c, the average storage cost C_{st} is given by

$$C_{st} = S_c \times ST \times (T/2) + S_c \times ST_0 \times T$$
$$= 182.5 \times S_c \times ST \times (ST + 2.0 \times ST_0)/TDT \qquad (3\text{-}11)$$

where the relation $ST = T \times (TDT/365)$ is used in the mathematical manipulation, and ST_0 is the emergency storage.

The storage cost per day per tonnage, S_c, itself depends on the construction costs of tanks, etc. and the maintenance expenses. Thus S_c is a function of the ship size
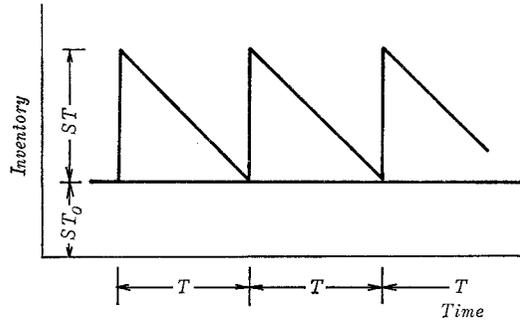
Fig. 4 Variation in inventory

and the emergency storage. Here, a simple model to S_c is adopted:

$$S_c = C_{s1} \times (ST + ST_0) + C_{s0} \qquad (yen/day/ton) \tag{3-12}$$

where C_{s1} and C_{s0} are the given constants.

When the fleet consists of different types of thips, ST in Eq. (3-11) should be replaced by the maximum transport capacity of the fleet. However, for the simplicity of the analysis, the formula (3-11) is used in the following even for the fleet consisting of different types of ships. Hence, the storage costs result in the lower estimation for the ships other than the ship having the maximum transport capacity.

4. Mathematical Models of Restrictions

4.1 Types of ships and transport capacity

The restrictions on the dead weight and the service speed of the ships are assumed to be given as follows:

$$DW_{min} \leqq DW \leqq DW_{max} \tag{4-1}$$

$$V_{min} \leqq V \leqq V_{max} \tag{4-2}$$

where DW_{min}, DW_{max}, V_{min} and V_{max} are the given constants.

Taking account of the deviation of the transport demand, some allowances are assumed to be given to the transport capacity of the fleet. The annual transport capacity of the fleet, SST, is given by

$$SST = \sum_{i=1}^{m} ST_i \times NR_i \times NS_i \tag{4-3}$$

where ST_i, NR_i and NS_i are the transport capacity per trip, the number of trips per year and the number of the ship type i, and m is the number of the types of the ships. Thus the allowances in the annual transport capacity are given in the form:

$$(1-\alpha_1) \leqq SST/TDT \leqq (1+\alpha_2) \tag{4-4}$$

where α_1 and α_2 are the given constants.

4.2 Draught limits

Consider the case where there are two routes between two ports and the route selection is made based on the dead weight and the loading conditions. Denoting the short cut and detour lengths of voyage by LV_1 and LV_2, the lengths of voyage at

the full and ballast load conditions, FLV and BLV, are categorized as follows:

$$FLV=LV_1, \ BLV=LV_1 \ \text{for} \ DW_{min}\leqq DW<DW_f \qquad (4\text{-}5)$$

$$FLV=LV_2, \ BLV=LV_1 \ \text{for} \ DW_f\leqq DW<DW_b \qquad (4\text{-}6)$$

$$FLV=LV_2, \ BLV=LV_2 \ \text{for} \ DW_b\leqq DW\leqq DW_{max} \qquad (4\text{-}7)$$

where DW_f and DW_b are the limit dead weights for both full and ballast load conditions.

5. Solution to Problem

5.1 Algorithmic procedure

First, restate the problem mentioned in Section 2:

ORIGINAL PROBLEM "Given the annual transport demand, TDT, find the types of ships $(DW_i, \ V_i)$ and number of them (N_i) to minimize the average annual total transport costs, H_c, considering the restrictions given in Section 4."

The problem is a nonlinear programming problem. However, the difficulty of the problem lies in the fact that the number of ships are not known and it is a control variable to be optimized. Here, an algorithmic procedure is proposed using the concept of dynamic programming.

For the application of dynamic programming, the annual total transport demand, TDT, is divided into N_d parts, where N_d is an appropriately selected number. From the upper part of TDT, those are denoted as the 1-st part, the 2-nd part, $\cdots$, the N_d-th part, respectively, as illustrated in Fig. 5. The minimum transport demand is given by

$$DDT=TDT/N_d \qquad (5\text{-}1)$$
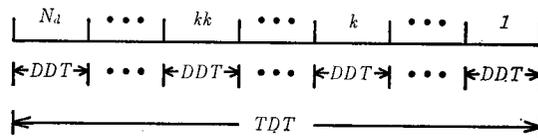
Fig. 5 Division of total annual transport demand

From this, it is seen that the number of the parts, N_d, must be selected, considering the compromise between the requirements on the optimization accuracy and the computer processing time. The original problem is reduced to the problem to determine the optimum type of ship $(DW, \ V)$ to carry the parts of TDT, which is given in the form to carry from one part to another part.

For the solution of the problem, the principle of optimality in dynamic programming[9] is applied: "For any k $(1\leqq k\leqq N_d)$, let the parts of TDT, i. e., from the first part to the $(k\text{-}1)$-th part, have been carried by some fleet. Whatever the number of parts carried by any type of ship starting from the k-th part may be, the subsequent parts of TDT must be transported by the optimum types of ships."

The solution to the original problem can be obtained by solving the following

two subproblems:

SUBPROBLEM 1 "Find the optimum type of ship, i. e., DW° (k,kk) and V° (k,kk), to carry the k-th to kk-th parts by a single ship, considering the restrictions given in Section 4."

SUBPROBLEM 2 "Find the optimum fleet to carry the k-th to N_d-th parts, considering the restrictions given in Section 4."

Subproblem 1 is to determine the optimum type of ship to carry the transport demand $DT = (kk-k+1) \times DDT$ by a single ship, and thus can be solved by using a nonlinear programming technique.

Subproblem 2 is solved as follows. The minimum transport cost carrying the kk-th to N_d-th parts by the optimum fleet and the minimum transport cost carrying the k-th to kk-th parts by the optimum type of ship are denoted by $H^\circ_c(kk)$ and $H_c(k,kk)$, respectively. It should be noted here that $H_c(k,kk)$ can be obtained by solving subproblem 1. Thus, from the principle of optimality, the minimum transport costs carrying the k-th to N_d-th parts are given by

$$H^\circ_c(k) = \min_{k \le kk \le N_d} [H_c(k,kk) + H^\circ_c(kk+1)] \qquad (5\text{-}2)$$

where $H^\circ_c(N_d+1) = 0$. The optimum value of kk is denoted by $kk(k)$.

Using the recurrence formula (5-2), the solution to the original problem can be obtained by sequentially solving subproblem 2 from $k = N_d$ to $k = 1$. This fact is easily proved by mathematical induction.

5.2 Computational considerations

For the economy of computation, the following remarks should be taken into account.

(1) The minimum and maximum amounts of cargo which a single ship can carry per year is calculated, and thus optimization in subproblem 1 has only to be made on the range

$$DT_{min} \le (kk-k+1) \times DDT \le DT_{max} \qquad (5\text{-}3)$$

where DT_{min} and DT_{max} are the minimum and maximum amounts of cargo which a single ship in the candidate fleet can transport per year.

(2) Since the optimum type ship which carries a specified amount of cargo is needed for the optimization of the fleet, the single sweep solution to subproblem 1 in the range given by Eq. (5-3) and storing the results in the memory are sufficient for the optimization followed.

Considering the above mentioned remarks, an algorithmic procedure is illustrated in Fig. 6 for a hypothetical example, where $N_d = 6$, $DT_{min} = DDT$, $DT_{max} = 4 \times DDT$. The arrows in the figure designate the optimum solutions for the case of carrying the corresponding part to the last part, i. e., the solutions to subproblem 2. On the heavy lines, the optimality principle is applied. Only for the cases designated by the number in the righthand side, the solution to subproblem 1 is needed because in any other cases the foregoing solutions are available. Thus, no new solutions to subproblem 1 are required for the cases where $k \le 2$ in this example.
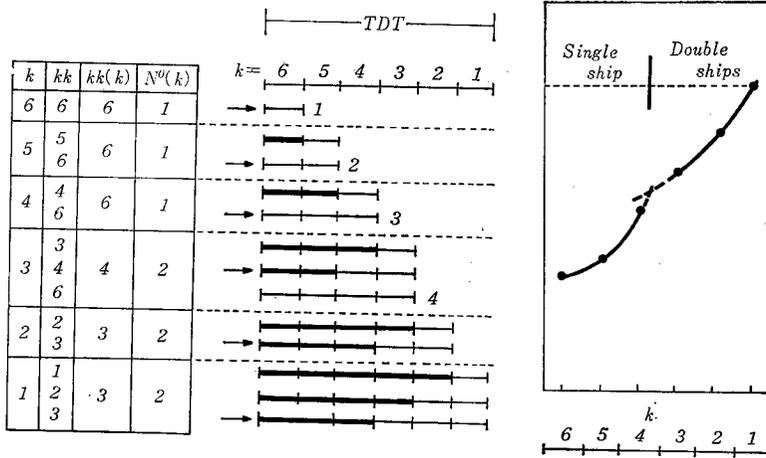
Fig. 6 Illustration of algorithmic procedure

Fig. 7 illustrates the optimization procedure for subproblem 1. For a given transport demand, DT, the candidate types of ships are searched in the three ranges, i. e., $DW_{min} \leqq DW < DW_f$, $DW_f \leqq DW < DW_b$ and $DW_b \leqq DW \leqq DW_{max}$, and comparing the

Fig. 7 Optimization procedure for subproblem 1

$k=N_d,\ k_p=0$
$DDT=TDT/Nd$
$H^s_c(N_d+1)=0$

START

$H^o_c(k)=10^{100}$

$k_1=kk-(k-1)$
$DT=k_1\times DDT$

$k_1\leq k_p$ Yes / No

Yes $DT<DT_{PASS}$ No

SOLVE SUBPROBLEM1

$H^o_c(k,kk)=H^s_c(k_1)$

$H^o_c(k_1)=H^s_c(k,kk)$
$DW_o(k)=DW^o(k,kk)$
$V_o(k_1)=V^o(k,kk)$
$k_p=k_1$

$H^o_c(k,kk)+H^o_c(kk+1)\leq H^o_c(k)$ No   Yes

$H^o_c(k)=H^o_c(k,kk)+H^o_c(kk+1)$
$kk_o(k)=kk$
$H^o_{cl}(k)=H^o_c(k,kk)$
$V_o(k)=V_o(k_1)$
$DW_{ol}(k)=DW_o(k_1)$

$kk\geq N_d$ Yes / No

$k\times DDT\leq DT_{MAX}$ No / Yes

$kk\geq N_d$ No / Yes

$kk=kk+1$
$DT=DT+DDT$

$\dfrac{N_d+k-1}{2}\geq kk$ Yes / No

$kk=N_d$
$DT=(N_d-k+1)\times DDT$

$DT\leq DT_{MAX}$ Yes / No

$I=1$
$N^o(k)=1$
$kk(I)=k-1$

$H^o_{cl}(I)=H^o_{cl}(kk(I)+1)$
$V_{ol}(I)=V_{ol}(kk(I)+1)$
$DW_{ol}(I)=DW_o(kk(I)+1)$
$kk(I)=kk(kk(I)+1)$

$I=I+1$
$N^o(k)=N^o(k)+1$ No $kk(I)\geq N_d$ Yes

$k=k-1$ No $k=1$ Yes   STOP

Fig. 8 Flow chart illustrating total optimization procedure

transport costs, the optimum type of ship is finally selected which gives the lowest transport costs.

Fig. 8 is a flow chart illustrating the total optimization procedure.

6. Numerical Examples

Consider the case where the following values are specified:

$DW_{min}=10000 dwt$, $D W_{max}=500000$ dwt

$V_{min}=13kt$, $V_{max}=19kt$, $\alpha_1=\alpha_2=0.1$

$LV_1=6600 n. m.$, $LV_2=13200$ $n. m.$, $c_{s0}=0$, $DDT=100000$ ton

The sequential unconstrained minimization technique ($SUMT$[10]) combined with the conjugate gradient method is applied for solving the nonlinear programming problem at each stage.

6.1 Link costs and transport capacity

In order to see the effect of the draught limits, the link costs and the transport capacity are calculated for the standard type ships with various DW dwt and $V=16$
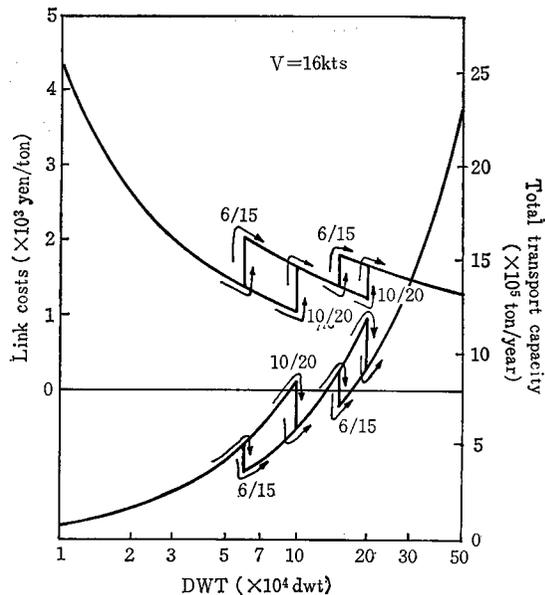

Fig. 9 Link costs and transport capacity of standard type ship

kt. Fig. 9 illustrates the results for two cases of $DW_f=60000$ $dwt/DW_o=150000$ dwt and $DW_f=100000$ $dwt/DW_b=200000$ dwt. It must be mentioned here that any tolls are not considered for both cases. From the figure, it is seen that the link costs per tonnage become lower as the types of ships become large. However, due to the detour, the dead weights corresponding to the full load limit give the minimum costs per tonnage for both cases. It should be noted here that the difference of the link costs between those of the full load limit, the ballast load limit and the maximum dead weight is very small for the former case.

6.2 Effect of total transport demand

The optimum fleets for various annual transport demands are listed in Table 3. The figures in the top rows correspond to the case of $DW_f=60000$ $dwt/DW_b=150000$ dwt and those in the bottom to that of $DW_f=100000$ $dwt/DW_b=200000$ dwt. The tolls and the storage costs are set to zero in the above calculations. In the former

Table 3 Effect of total transport demand on optimum fleet $(S_c = C_t = 0)$

TDT	DW	V	H_c	No	DW	V	H_c	No
5	1. 36 6. 77	15. 89 14. 33	3. 42 6. 71	1 1	5. 78	13. 39	5. 93	1
10	4. 25 2. 88	13. 89 13. 95	5. 16 4. 31	1 1	13. 81 9. 99	14. 1 15. 54	10. 39 8. 88	1 1
20	46. 48 9. 83	16. 00 14. 47	27. 26 8. 13	1 2	8. 14	14. 24	7. 42	1
30	14. 99 9. 33	17. 00 14. 47	12. 60 8. 13	1 2	49. 06 9. 99	15. 89 15. 54	28. 38 8. 88	1 2
40	45. 01 9, 99	15. 67 15. 54	26. 12 8. 88	1 5	49. 06	15. 89	28. 38	1
50	5. 78 2. 88	13. 39 13. 95	5. 93 4. 31	1 1	49. 99 9. 99	16. 98 15. 54	30. 85 8. 88	2 6

TDT : $\times 10^6$ *ton/year*, DW : $\times 10^4 dwt$, V :kt, H_c :$\times 10^8$ *yen/year*, No : number of ships
Top : $DW_f = 60000 \ dwt/DW_b = 150000 \ dwt$, Bottom : $DW_f = 100000 dwt/DW_b = 200000 \ dwt$

case, the various types of ships can be the constituents of the optimum fleet depending on the transport demand. This can be explained by the fact that the differnce of the link costs is very small between those of the full load limit, the ballast load limit and the maximum dead weight as pointed out in Section 6.1. On the contrary, only the types of ships below the full load limit are optimum for the latter case. This illustrates the fact that the small types of ships which can pass the short cut route is more economical than VLCC (Very Large Crude Carriers) in this case due to the detour. However, it should be noted that no tolls are assumed to be imposed on the ships passing the short cut route.

6.3 Effect of tolls

Using the model given in Section 3.1.3, the effect of tolls is discussed. As seen from Table 4, there arise the cases where the types of ships which can pass the short

Table 4 Effect of tolls on optimum fleet $(TDT = 5 \times 10^6 \ ton/year, \ S_c = 0)$

	C_t															
	0				100				300				600			
	DW	V	H_c	No	DW	V	H_c	No	DW	V	H_c	No	DW	V	H_c	No
6/15	5. 78 49. 99	13. 39 16. 98	5. 93 30. 85	1 2	5. 83 49. 99	13. 26 16. 98	6. 73 30. 85	1 2	5. 69 49. 99	13. 63 16. 98	8. 34 30. 85	1 2	17. 59 49. 06 49. 94	14. 98 15. 89 16. 40	12. 42 28. 38 29. 55	1 1 1
10/20	2. 88 9. 99	13. 95 15. 54	4. 31 8. 88	1 6	9. 99 18. 89	15. 54 15. 13	10. 48 14. 19	5 1	8. 14 49. 99	14. 24 16. 38	11. 02 29. 55	1 2	22. 23 47. 02 48. 67	15. 17 15. 79 16. 03	14. 78 27. 25 28.-39	1 1 1

6/15 : $DW_f = 60000 \ dwt/DW_b = 150000 \ dwt$, 10/20 : $DW_f = 100000 \ dwt/DW_b = 200000 \ dwt$
DW : $\times 10^4 \ dwt$, V : kt, H_c : $\times 10^8$ *yen/year*, No : number of ships, C_t : *yen/dwt/passage*

cut route only in the ballast concition and/or which must pass the detour route in either way become the constituent of the optimum fleet, depending on the tolls imposed on the ships passing the short cut route.

6.4 Effect of storage costs

In order to illustrate the effect of the storage costs on the optimum fleet, the storage cost coefficient, C_{s1}, is varied with $C_{s0}=0$ fixed, and the results are given in Table 5. As C_{s1} becomes large, the storage costs of the cargo carried by individual

Tableo 5 Effect of storage cost coefficient on optimum fleet ($TDT=5\times10^6$ $ton/year$, $DW_f=60000$ dwt, $DW_b=1500000$ dwt, emergency storage=tolls=0)

C_{s1}															
0				10^{-6}				10^{-4}				10^{-2}			
DW	V	H_c	No	DW	V	H_c	No	DW	V	H_c	No	DW	V	H_c	No
5.78	13.39	5.93	1	5.35	14.64	5.97	1	5.73	13.54	5.98	8	3.53	17.42	6.59	2
49.99	16.98	30.85	2	49.99	16.96	31.04	2	14.99	17.30	13.26	2	3.78	18.42	8.74	11

DW : $\times10^4$ dwt, V : kt, H_c : $\times10^8$ $yen/year$, No : number of ships, C_{s1} : $yen/day/ton^2$

ships of the fleet weigh relative to the other costs, and thus the small types of ships which give the low storage costs become optimum.

6.5 Effect of emergency storage

For the constant value of the storage cost coefficient, C_{s1}, the storage costs of the cargo transported by an individual ship become small relative to that of the emergency storage, ST_0, as ST_0 becomes large. Thus, the optimum fleet becomes of the slightly larger type ship, as shown in Table 6. The emergency storage in the table is expressed as the days during which the neccessary demand can be met without any supply.

6.6 Effect of the number of divisions on optimization results

Table 7 illustrates the effect of the number of divisions, N_d, on the optimization results. $N_d=57$ corresponds to the case where DDT is carried by the smallest standard

Table 6 Effect of emergency storage on optimum fleet ($TDT=5\times10^6$ $ton/year$, $DW_f=60000dwt$, $DW_b=150000dwt$, $C_{s1}=10^{-6}$ $yen/day/ton^2$, $C_t=0$)

0 $days$				30 $days$				90 $days$				180 $days$			
DW	V	H_c	No	DW	V	H_c	No	DW	V	H_c	No	DW	V	H_c	No
5.35	14.64	5.97	1	5.61	13.86	6.00	1	14.99	16.96	13.78	3	14.99	17.02	16.95	3
49.99	16.96	31.04	2	49.99	17.03	31.82	2	49.99	17.05	35.08	1	49.99	17.07	44.23	1

DW : $\times10^4 dwt$
V : kt
H_c : $\times10^8$ $yen/year$
No : number of ships

Table 7 Effect of number of divisions on optimization results.
($TDT=5\times10^6\ ton/year$, $S_e=0$, $C_t=300yen/dwt/passage$)

N_d																	
57						50						25					
DW	V	H_e	No	Cost	Time	DW	V	H_e	No	Cost	Time	DW	V	H_e	No	Cost	Time
3,78	13.56	6.38	1	1.0	1.0	5.69	13.63	8.34	1	1.01	0.94	11.96	14.41	11.22	1	1.02	0.46
49.99	16.93	31.42	2			49.99	16.98	30.85	2			49.99	16.38	29.55	2		

DW : $\times10^4\ dwt$
V   : kt
H_e : $\times10^9\ yen/year$
No : number of ships
Cost : ratio of total annual transport costs
Time : ratio of computer processing time

type ship, i. e., $DW=10000\ dwt$ and $V=16\ kt$, to be considered. In the columns "cost" and "time" of the table are listed the ratios of the total annual transport costs and the computer processing time of each case to the case $N_d=57$. As N_d becomes large, the computation time becomes large, while the optimization results are improved. Thus, the value of N_d should be selected, considering the compromise between the optimization accuracy and the computer processing time.

7. Conclusions

The problem is considered for determining the optimum fleet-size to minimize the transport costs for a given transport demand, using the systems engineering techniques. The mathematical models relating the transport costs to a fleet-size have been developed. The algorithmic procedure applying the principle of optimality in dynamic programming and nonlinear programming techniques is given for the solution of the problem. Using the software program, the effects of the transportation system's factors, such as the transport demand, the tolls, the storage costs, etc., have been discussed both quantitatively and qualitatively. Many works are still left to be done relating the present work, e. g., improvement of the software program and the mathematical models, sensitivety analysis of the optimum fleet, selection of the optimality criterion, forecasting the transport demand and its treatment, etc..

Acnowledgements

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