Influence upon Power Flow by Changing Taps of Transformer

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Influence upon Power Flow by Changing Taps of Transformer

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In this paper, the authors suggest about the treatment of tap changing transformer on the power system.

In changing the taps of transformer, the leakage impedance in its winding varies as a function of off-nominal turn-ratio.

In power flow analysis, it seems that the variation of leakage impedance of transformer has not been included in the calculation as yet.

Then, the authors study out a method of treatment of tap changing transformer in the case of considering the variation of leakage impedance, and clarify the difference of the influence upon the system performances between this and the former cases.

1. Introduction

The equivalent circuit of tap changing transformer for the calculation of power flow has been given by J.B.Ward and H.W.Hale.[1]

But, in this equivalent circuit, as the variation of leakage impedance by the change of taps of transformer is not considered, this circuit has to be modified, to be exact.

Generally, in changing the taps of transformer, the leakage impedance in its winding varies as a function of off-nominal turn-ratio.

Then, the authors derive out a modified equivalent circuit, assuming tentatively that the leakage impedance varies in proportion to off-nominal turn-ratio squared though it must be decided according to the design of tapping winding.[2]

The influence upon the power system by the variation of leakage impedance of transformer depends upon its power system form.

If the influence described above is weak, it is not necessary to consider the variation of leakage impedance, and it may be neglected.

But, on the system, line constants of which are comparable with the leakage impedance of transformer, its system will be considerably affected by the variation of leakage impedance.

In order to elucidate the above facts, the authors investigated the influence upon the system voltage-reactive power flow control by the variation of leakage impedance of transformer.

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2. Treatment of tap changing transformer

(a) Equivalent circuit of tap changing transformer

The transformer in Fig.2-1 has been usually represented by the equivalent circuit as shown in Fig.2-2 (see appendix 1), where

n: off-nominal turn-ratio,

$\dot{Z}_t$: leakage impedance under nominal condition in per unit.

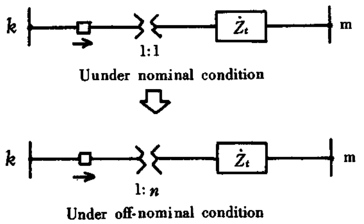


Fig. 2-1

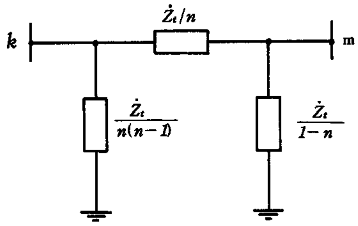


Fig. 2-2

The advantage of this equivalent circuit is that all other branch impedances except k-m one need not be corrected when the tap is changed, and the values of electrical quantities in per unit can be converted to the physical values in practical unit, keeping the transformation ratio under nominal condition intact in spite of the change of taps.

But, in the above treatment of tap changing transformer, the variation of leakage impedance by the change of taps is not included in the calculation.

Assuming that the leakage impedance varies in proportion to off-nominal turn-ratio squared, the leakage impedance under off-nominal condition $\dot{Z}_{tn}$ is given by the following relation:

$$\dot{Z}_{tn} \doteqdot n^2 \dot{Z}_t \quad \cdots\cdots\cdots(2\text{-}1)$$

According to Eq.(2-1), the transformer under off-nominal condition in Fig.2-3 can be represented by the equivalent circuit as shown in Fig.2-4.

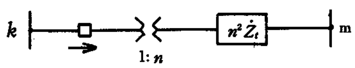


Fig. 2-3

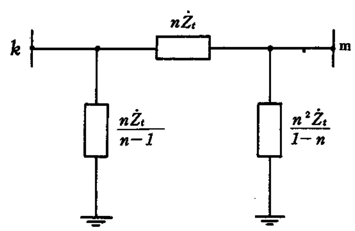


Fig. 2-4

(b) Consideration by an example

One concrete example will be given to help to the explanation done so far.

Example circuit is shown in Fig. 2-5, where

$\dot{Z}_{12}$: tap changing tranformer leakage impedance viewed from secondary circuit,
Z_{23} : transmission line impedance

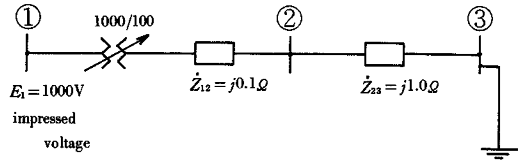
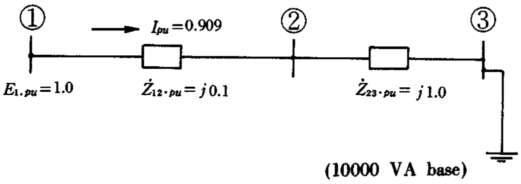


Fig. 2-5

Per unit representation of Fig. 2-5 is shown in Fig. 2-6.



(10000 VA base)

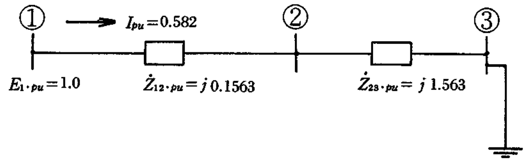
Fig. 2-6

Assuming that the number of turns of secondary winding of transformer changed from 100 turns to 80 turns, the off-nominal turn-ratio becomes 0.8, and, referring to Eq.(2-1), the value of leakage impedance becomes 0.064Ω .

Case (1) When the variation of leakage impedance by the change of taps is not included in the calculation.

In this case, in spite of the change of taps, the value of leakage impedance is held 0.1Ω and the circuit in Fig.2-6 is modified as shown in Fig.2-7.

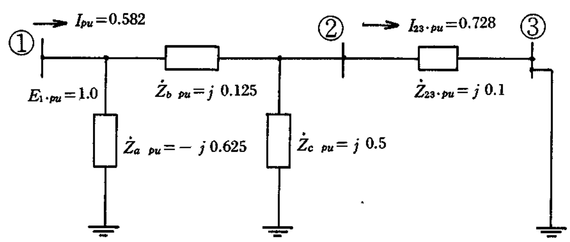
Then, the impedance of branch ② - ③ will be changed from 1.0 to 1.563 in per unit.



(10000 VA base)

Fig. 2-7

Remaining the impedance of branch ② ③ as 1.0 in per unit, the circuit in Fig.2-7 can be substituted by the equivalent circuit as in Fig.2-8 on reference ot Fig.2-2.



(10000 VA base)

Fig. 2-8

In Fig.2-7 and Fig.2-8, the currents in practical unit are calculated as shown in table 2-1.

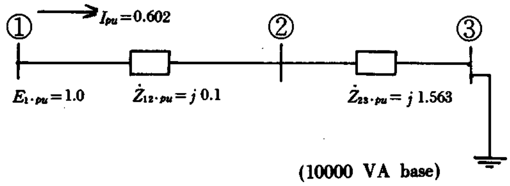
Table 2−1

(*Base capacity* : 10000 *VA*)

	The current in 1−ry 1000 V side	The current in 2−ry 80 V side
Fig. 2−7	$0.582 \times \frac{10000}{1000} = 5.82\ A$	$0.582 \times \frac{10000}{80} = 72.8\ A$
Fig. 2−8	$0.582 \times \frac{10000}{1000} = 5.82\ A$	$0.782 \times \frac{10000}{100} = 72.8\ A$ (*Keeping the nominal voltage 100 V intact*)

Case (2) When the variation of leakage impedance by the change of taps is included in the calculation

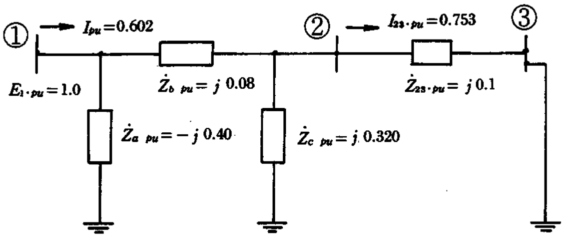
In this case, the circuit is shown in Fig.2-9 in per unit system.



(10000 VA base)

Fig. 2−9

Remaining the impedance of branch ② - ③ as 1.0 in per unit, the circuit in Fig.2-9 can be substituted by the equivalent circuit as in Fig.2-10 on reference to Fig.2-4.



(10000 VA base)

Fig. 2−10

In Fig.2-9 and Fig.2-10, the currents in practical unit are calculated as shown in Table 2-2.

Table 2−2

(*Base capacity* : 10000 *VA*)

	The current in 1−ry 1000 V side	The current in 2−ry 80 V side
Fig. 2−9	$0.602 \times \frac{10000}{1000} = 6.02\ A$	$0.602 \times \frac{10000}{80} = 75.3\ A$
Fig. 2−10	$0.602 \times \frac{10000}{1000} = 6.02\ A$	$0.753 \times \frac{10000}{100} = 75.3\ A$ (*Keeping the nominal voltage 100 V intact*)

There is some difference in the results between this case(2) and aforementioned case(1).

As the proportion of the impedance of transformer included in the line impedance becomes larger as compared with the line impedance, this difference increases and may not be neglected.

From the above fact, it may be said that the variation of leakage impedance by the change of taps has to be included in the calculation and the transformer had better to be represented by the equivalent circuit as shown in Fig.2-4.

3. Influence upon voltage-reactive power flow control by variation of leakage impedance of tap changing transformer

The methods of control for the system voltage-reactive power are shown in Table 3–1[3].

Table 3–1

Controlled system \ Controlled variable		Reactive power: Peak load hour	Reactive power: Night	Voltage: Peak load hour / Night
A	275/77 KV substation, possesing thermal power plant in 77 KV system.	To compensate reactive power loss of line, transformer and a part of reactive load by the capacitor (S. C).	To absorb the charging capacity by main line, etc., by the shunt reactor (S. R).	To keep nominal voltage by load ratio controller following after reactive power control.
B	275/77 KV substation, having load only.	The same as the above.	The same as the above.	The same as the above.
C	275/77 KV substation, near by 275 KV thermal power plant.	To supply reactive power 275 KV thermal power plant.	Condensive driving of 275 KV thermal power plant generator.	To keep voltage on the 77 KV basic curve by load ratio controller.
D	275 KV tie point.	Under deliberation among utility officials, to abide by the agreed P–Q pattern.	The same as the left.	To aim the voltage required for the coordination of system operation.

Case(A) Example system of Case(A) is show in Fig. 3–1, where

$\dot{V}_S$: infinite bus voltage,

$\dot{V}_r$: thermal power plant bus voltage,

$\dot{V}$: secondary bus voltage of load ratio control transformer,

x_t : load ratio control transformer leakage impedance in nominal tap,

n : off-nominal turn-ratio,

x : transmission line impedance,

P_s : sending acitve power,

Q_t, q : reactive power.

and in this systme, the resistances of transformer and transmission line are neglected.

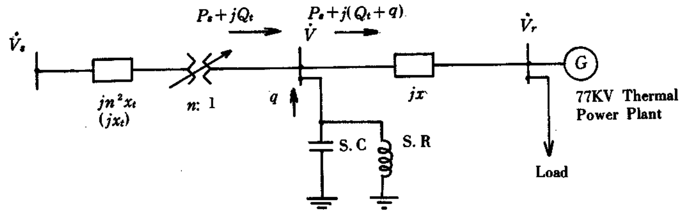


Fig. 3–1

(1) When the variation of leakage impedance is included in the calculation

The following equations are formulated:

$$\left.\begin{aligned}&(Q_t n^2 x_t + n^2 V^2)^2 + (P_S n^2 x_t)^2 = V^2 n^2 V_S^2\\&\left\{(Q_t+q)x - V^2\right\}^2 + (P_S x)^2 = V^2 V_r^2\end{aligned}\right\} \quad (3\text{–}1)$$

Under the proper approximation, that is, omission of the relatively small quantities in the equations or putting lastly all of the values of V_S, V and V_r at 1.0 in per unit, etc., through the use of partial derivatives of Eqs. (3–1) with respect to V, the following relations can be obtained (see appendix 2);

$$\left.\begin{aligned}&\Delta V = A_n \Delta n + A_q \Delta q + A_{Vr} \cdot \Delta V_r\\&\Delta Q_t = B_n \Delta n + B_q \Delta q + B_{Vr} \Delta V_r\end{aligned}\right\} \quad (3\text{–}2)$$

where $\Delta V, \Delta Q_.$: controlled variables,

$\Delta n, \Delta q, \Delta V_{r.}$: manipulating variables,

$A_n, A_q, A_{Vr}, B_n, B_q$ and B_{Vr}: so called system sensitivity constants and become as shown in Table 3–2 in this case.

Table 3–2

A_n	$-\dfrac{1/n\cdot(2n^2-1)x}{n^2x_t+x(2n^2-1)}$	B_n	$-\dfrac{1/n\cdot(2n^2-1)}{n^2x_t+x(2n^2-1)}$
A'_n	$-\dfrac{1/n\cdot(2n^2-1)x}{x_t+x(2n^2-1)}$	B'_n	$-\dfrac{1/n\cdot(2n^2-1)}{x_t+x(2n^2-1)}$
A_q	$-\dfrac{xn^2x_t}{n^2x_t+x(2n^2-1)}$	B_q	$-\dfrac{x(2n^2-1)}{n^2x_t+x(2n^2-1)}$
A'_q	$-\dfrac{xx_t}{x_t+x(2n^2-1)}$	B'_q	$-\dfrac{x(2n^2-1)}{x_t+x(2n^2-1)}$
A_{Vr}	$\dfrac{n^2x_t}{n^2x_t+x(2n^2-1)}$	B_{Vr}	$\dfrac{(2n^2-1)}{n^2x_t+x(2n^2-1)}$
A'_{Vr}	$\dfrac{x_t}{x_t+x(2n^2-1)}$	B'_{Vr}	$\dfrac{(2n^2-1)}{x_t+x(2n^2-1)}$

(2) When the variation of leakage impedance is not inculded in the calculation

The following equations are formulated :

$$\left.\begin{aligned}&(Q_t x_t + n^2 V^2)^2 + (P_S x_t)^2 = V^2 n^2 V_S^2\\&\left\{(Q_t+q)x - V^2\right\}^2 + (P_S x)^2 = V^2 V_r^2\end{aligned}\right\} \quad (3\text{–}3)$$

Using the same methods as the case(1), the system sensitivity constants, A'_n, A'_q A'_{Vr}, B'_n, B'_q and B'_{Vr} can be obtained as shown in Table. 3–2.

Case (B) Example system of Case (B) is shown in Fig. 3–2, where

V_L bus voltage at load point,

other symbols : the same as in Fig. 3–1.

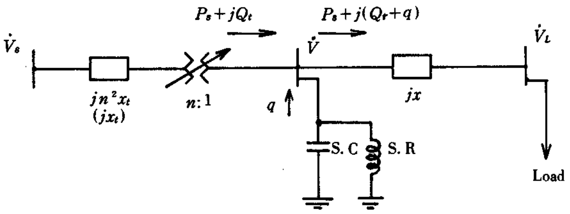


Fig. 3-2

Relations among controlled and manipulating variables are as follows:

$$\left.\begin{aligned} \Delta V &= A_n \Delta n + A_q \Delta q + A_{VL} \Delta V_L = A_n \Delta n + A_q \Delta q + A_{QL} \Delta Q_L \\ \Delta Q_t &= B_n \Delta n + B_q \Delta q + B_{VL} \Delta V_L = B_n \Delta n + B_q \Delta q + B_{QL} \Delta Q_L \end{aligned}\right\} \quad (3\text{-}4)$$

where, $\dfrac{\Delta V_L}{\Delta Q_L} = K, \quad A_{QL} = K A_{VL}, \quad B_{QL} = K B_{VL}$

K: a constant decided by the system and load characteristics (see appendix 3) and all other system sensitivity constants in this cae are quite the same as those shown in Table 3–2 of Case(A), except that $A_{Vr}, A'_{Vr}, B_{Vr},$ and B'_{Vr} are replaced with A_{VL}, A'_{VL}, B_{VL} and B'_{VL}, respectively.

Case(C) Example system of Case(C) is shown in Fig. 3–3, where

$\dot{V}_S$: 275 KV thermal power plant bus voltage,

$\dot{V}$: secondary bus voltage of load ratio control transformer,

x_t : load ratio control transformer leakage impedance in nominal turn-ratio

n : off-nominal turn-ratio.

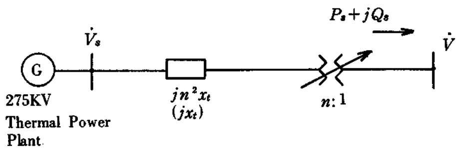


Fig. 3–3

Relation among controlled and manipulating variables is as follows :

$$\Delta V = A_n \Delta n + A_{QS} \Delta Q_S + A_{VS} \Delta V_S \quad (3\text{-}5)$$

On reference to the same method as the aformentioned, the system sensitivity constants in this case are obtained as shown in Table 3–3.

Table 3–3

A_n	$-\dfrac{1/n}{2n^2-1}$
A'_n	$-\dfrac{1/n}{2n^2-1}$
A_{Qs}	$-\dfrac{n^2 x_t}{2n^2-1}$
A'_{Qns}	$-\dfrac{x_t}{2n^2-1}$
A_{VS}	$\dfrac{1}{2n^2-1}$
A'_{VS}	$\dfrac{1}{2n^2-1}$

Case(D) Example system of Case (D) is shown in Fig. 3–4, where

$\dot{V}_0$, $\dot{V}_h$: infinite bus voltages,

$\dot{V}_1$: ideal transformer primary voltage

n : off-nominal turn-ratio,

x_t : load ratio control transformer leakage impedance in nominal tap,

x_1, x_2 : transmission line impedances.

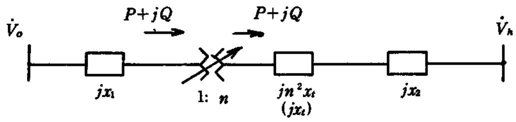


Fig. 3–4

Relations among conrtolled and manipulating vairables are as follows:

$$\left.\begin{aligned}\Delta V_1 &= A_n \Delta n\\ \Delta Q &= B_n \Delta n\end{aligned}\right\} \qquad (3\text{–}6)$$

The system sensitivity constants in this case are obtained similarly as shown in Table 3–4.

Table 3–4

A_n	$\dfrac{1/n\cdot(2/n^2-1)(n^2x_t+x_2)}{x_1+(n^2x_t+x_2)(2/n^2-1)}$	B_n	$\dfrac{1/n\cdot(2/n^2-1)}{x_1+(n^2x_t+x_2)(2/n^2-1)}$
A'_n	$\dfrac{1/n\cdot(2/n^2-1)(x_t+x_2)}{x_1+(x_t+x_2)(2/n^2-1)}$	B'_n	$\dfrac{1/n\cdot(2/n^2-1)}{x_1+(x_t+x_2)(2/n^2-1)}$

The above mentioned are the detailed comparision of the system sensitivity constants of both cases, that is, the variation of leakage impedance of transformer is included in the calculation or not.

Now, as an example of each case, the authors assume that, in the respective system of the above cases(A)~(D), the line impedances are nearly equal to the leakage impedance of transformer, and consider about the cases of four kinds of tap positions of transformer (a, b, c and d), in which the corresponding values of voltage regulation and off-nominal turn-ratios are as shown in Table 3–5.

Table 3–5

Tap position	*a*	*b*	*c*	*d*
Voltage regulation value (%)	−10	− 7.5	+ 7.5	+10
n	0.90	0.925	1.075	1.10

The results are shown in Table 3–6 ~ Table 3–8.

Table 3–6

	$\frac{A_n}{A'_n}$	$\frac{A_q}{A'_q}$	$\frac{A_{Vr}}{A'_{Vr}}$	$\frac{B_n}{B'_n}$	$\frac{B_q}{B'_q}$	$\frac{B_{Vr}}{B'_{Vr}}$
a	1.132867	0.917622	0.917622	1.132867	1.132867	1.132867
b	1.092142	0.934463	0.934463	1.092142	1.092142	1.092142
c	0.936914	1.082721	1.082721	0.936914	0.936914	0.936914
d	0.920152	1.113384	1.113384	0.920152	0.920152	0.920152

Note : In case (B), the results' are quit the same as in case (A), except that A_{Vr}, A'_{Vr}, B_{Vr} and B'_{Vr} are replaced with A_{QL}, A'_{QL}, B_{QL} and B'_{QL} .

Table 3–7

	$\frac{A_n}{A'_n}$	$\frac{A_{QS}}{A'_{QS}}$	$\frac{A_{VS}}{A'_{VS}}$
a	1.0	0.810	1.0
b	1.0	0.855625	1.0
c	1.0	1.155625	1.0
d	1.0	1.210	1.0

Table 3–8

	$\frac{A_n}{A'_n}$	$\frac{B_n}{B'_n}$
a	0.974037	1.076284
b	0.979267	1.055458
c	1.030217	0.943875
d	1.042981	0.955841

From these resuts, it became clear that the difference of estimated controlled variables between both cases may become more than several per-cent according to the system conditions.

4. Conclusion

The larger the value of voltage regulation and the rate of occupation of the leakage impedance of transformer in a power system, the larger the influence upon the system performances by the variation of the leakage impedance is. It is clear, as aleady mentioned, from the results shown in Table 3–6 ~ Table 3–8.

Therefore, in the system analysis, the variation of leakage impedance of transformer by the change of taps may not be ignored, and the equivalent π circuit shown in Fig. 2–4 will be validly used.

Appendices

(1) Derivation of equivalent circuit of tap changing transformer

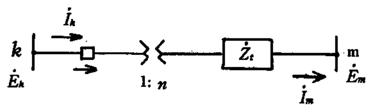


Fig. A-1

In Fig. A-1, the following equation is formularized :

$$\begin{bmatrix}\dot{E}_k\\ \dot{I}_k\end{bmatrix}=\begin{bmatrix}\frac{1}{n} & 0\\ 0 & n\end{bmatrix}\begin{bmatrix}1 & \dot{Z}_t\\ 0 & 1\end{bmatrix}\begin{bmatrix}\dot{E}_m\\ \dot{I}_m\end{bmatrix}=\begin{bmatrix}\frac{1}{n} & \dot{Z}_t/n\\ 0 & n\end{bmatrix}\begin{bmatrix}\dot{E}_m\\ \dot{I}_m\end{bmatrix} \quad (A\text{-}1)$$

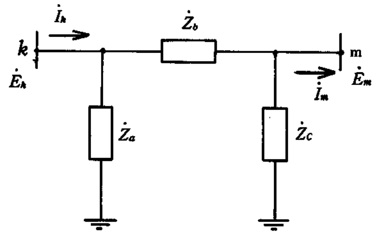


Fig. A-2

In Fig. A-2, the following equation is formularized :

$$\begin{bmatrix}\dot{E}_k\\ \dot{I}_k\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1/\dot{Z}_a & 1\end{bmatrix}\begin{bmatrix}1 & \dot{Z}_b\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1/\dot{Z}_c & 1\end{bmatrix}\begin{bmatrix}\dot{E}_m\\ \dot{I}_m\end{bmatrix}$$

$$=\begin{bmatrix}1+\dot{Z}_b/\dot{Z}_c & \dot{Z}_b\\ 1/\dot{Z}_a+\dot{Z}_b/\dot{Z}_a\dot{Z}_c+1/\dot{Z}_c & 1+\dot{Z}_b/\dot{Z}_a\end{bmatrix}\begin{bmatrix}\dot{E}_m\\ \dot{I}_m\end{bmatrix} \quad (A\text{-}2)$$

In order that the circuit of Fig. A-1 is equivalent to the circuit of Fig. A-2, the following equation must be satisfied :

$$\begin{bmatrix}1/n & \dot{Z}_t/n\\ 0 & n\end{bmatrix}=\begin{bmatrix}1+\dot{Z}_b/\dot{Z}_c & \dot{Z}_b\\ 1/\dot{Z}_a+\dot{Z}_b/\dot{Z}_a\dot{Z}_c+1/\dot{Z}_c & 1+\dot{Z}_b/\dot{Z}_a\end{bmatrix} \quad (A\text{-}3)$$

Solving the Eq, A-3 for $\dot{Z}_a, \dot{Z}_b$ and $\dot{Z}_c$ the next relations can be obtained

$$\dot{Z}_a=\dot{Z}_t/n(n-1)$$
$$\dot{Z}_b=\dot{Z}_t/n$$
$$\dot{Z}_c=\dot{Z}_t/(1-n)$$

(2) Derivation of Eq. 3-2[4)]

Differentiating partially Eqs. 3-1 with respect to V, Eqs. 3-1 are expanded as follows :

$$\left.\begin{aligned}&(2Q_t n^2 x_t^2+2x_t n^4 V^2)\frac{\partial Q_t}{\partial V}+(8Q_t x_t n^3 V^2+2Q_t^2 n x_t^2+4n^3 V^4-2nV^2V_S^2)\frac{\partial n}{\partial V}\\&=2n^2 V V_S^2-4Q_t x_t n^2 V-4n^4 V^3\\&(2Q_t x^2+2qx^2-2xV^2)\frac{\partial Q_t}{\partial V}+(2Q_t x^2+2qx^2-2xV^2)\frac{\partial q}{\partial V}-2V^2V_r\frac{\partial V_r}{\partial V}\\&=2VV_r^2+4xVQ_t+4xVq-4V^3\end{aligned}\right\}\quad (A\text{-}4)$$

Neglecting the product of reactive power and reactance as it is small as compared with other terms, and considering that all of the values of V, V_S and V_r are approximately 1.0, Eqs. A–4 is transformed as follows :

$$\left.\begin{aligned}&\frac{\partial Q_t}{\partial V}=-\frac{2n^2-1}{n^3 x_t}\frac{\partial n}{\partial V}-1/n^2\cdot x_t(2n^2-1)\\&\frac{\partial Q_t}{\partial V}=-\partial q/\partial V-1/x\cdot\partial V_r/\partial V+1/x\end{aligned}\right\}\quad (A-5)$$

The following equations are obtained on reference to Eqs. A–5 :

$$\Delta V=A_n\Delta n+A_q\Delta q+A_{Vr}\Delta V_r$$
$$\Delta Q=B_n\Delta n+B_q\Delta q+B_{Vr}\Delta V_r$$

(3) Explanation of constant K

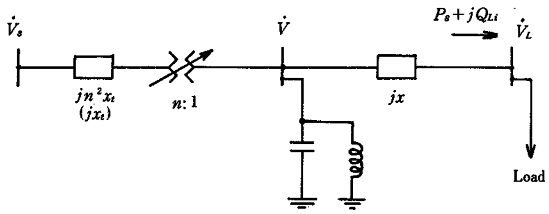


Fig. A–3

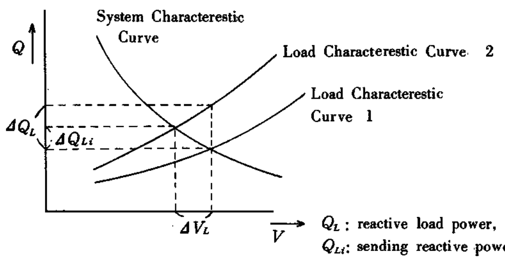


Fig. A–4

Q_L : reactive load power,
Q_{Li}: sending reactive power,
k_L : compound load characterestic sensitivity

At the point of V_L, the following equation is formularized :

$$(Q_{Li}x+V_L^2)^2+(xP_S)^2=V_L^2V^2 \quad (A\text{-}8)$$

By the same methods aforementioned, from the Eq. A–8, the following system characteristic sensitivity is obtained :

$$\frac{\Delta Q_{Li}}{\Delta V_L}=\frac{\Delta V/\Delta V_L-1}{x}=A \quad (A\text{-}9)$$

From Fig. A–4, the next relation holds approximately between the change of reactive load, $\varDelta Q_L$, and the change of voltage at load point, $\varDelta V_L$,

$$\varDelta Q_L + k_l\ \varDelta V_L = A\varDelta V_L \qquad (A\text{–}10)$$

dividing both members of Eq. A–10 by $\varDelta V_L$,

$$\varDelta Q_L / \varDelta V_L + k_l = A$$

and rearranging this equation, a constant K in this paper is difined as follows :

$$K = \varDelta Q_L / \varDelta V_L = A - k_l \qquad (A\text{–}11)$$

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