

The Stress Distribution around a Hole in the Web of Beam Subjected to Pure Bending

著者	Okamura Yoichiro, Okada Hiroo, Fukumoto Yoshio
引用	Bulletin of University of Osaka Prefecture. Series A, Engineering and natural sciences. 1967, 16(1), p.159-170
URL	http://doi.org/10.24729/00008893

The Stress Distribution around a Hole in the Web of a Beam Subjected to Pure Bending

Yoichiro OKAMURA*, Hiroo OKADA* and Yoshio FUKUMOTO*

(Received June 15, 1967)

The stress analysis of perforated beam with flanges subjected to pure bending is studied. The calculation is executed for three typical cases and results of numerical calculation are shown. Moreover, the theory is verified by experiments.

1. Introduction

The holes in a loaded beam will affect the stress distribution in the beam, especially in the neighbourhood of the holes.

While many studies on the stress distribution in a perforated strip have been executed, the effect of the flanges attached to both sides of the perforated strip on the stress distribution is not systematically evaluated.

In general, the beams are simultaneously submitted to the action of tensile or compressive force, bending moment and shearing force. In such case, the stresses in the beam can be obtained by superposing the stresses produced by each of these loads. The solution of tension problem was already reported by the authors¹⁾.

In this paper the stress analysis of the perforated beam under pure bending is studied, by using the same method as in the tension problem and the effect of flange of the beam on the stress distribution are clarified. Moreover, the theoretical results are compared with the experimental.

It may be added that when hole is small the maximum stress in the beam under pure bending occurs at the extreme fiber. And the maximum value of the stress in this case is inferred to be equal to the value of nominal bending stress at the extreme fiber in the minimum section, from the published data²⁾ regarding the perforated strip with no flange. Therefore numerical calculation here is mainly executed for the stresses around the hole.

2. Theory

We consider the perforated strip which has one circular hole mid-way between the edges and is stiffened symmetrically with the flanges along both edges. (Fig. 1) The width and the thickness of the perforated strip and of the flange are $2t_w$ and $2b$, t_f , respectively;

* Department of Naval Architecture, College of Engineering.

the radius of the circular hole is λ . This perforated strip with the flanges is submitted to the action of the pure bending moment which produces the bending stress T at the junction of the perforated strip and the flanges at infinity. Both the perforated strip and

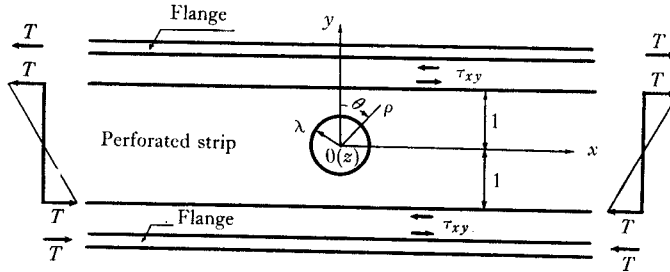


Fig. 1.

the flanges are regarded as thin plates and the normal stresses perpendicular to the plane of the flange are ignored.

2-1. Stress Function of the Perforated Strip (Successive Approximation)

As shown in Fig. 1, Cartesian co-ordinates (x, y, z) and polar co-ordinates (ρ, θ) will be used. It will be convenient to take the initial line along the y -axis and the positive direction of θ clockwise.

Denoting the stress function χ , χ must satisfy the equation

$$\frac{\partial^4 \chi}{\partial x^4} + 2 \frac{\partial^4 \chi}{\partial x^2 \partial y^2} + \frac{\partial^4 \chi}{\partial y^4} = 0 \tag{1}$$

and the following condition (a), (b) and (c):

(a) At infinity $x = \pm \infty$,

$$\sigma_x = \frac{\partial^2 \chi}{\partial y^2} = Ty, \quad \sigma_y = \frac{\partial^2 \chi}{\partial x^2} = 0, \quad \tau_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y} = 0$$

(b) On the straight edges $y = \pm 1$,

$$\sigma_y = 0 \quad \text{and} \quad \epsilon_x = \bar{\epsilon}_x$$

where ϵ_x is the normal strain in the x -direction and $\bar{\epsilon}_x$ is the normal strain occurring in the flange at the line of connection between the flanges and the perforated strip.

(c) On the edge of the hole $\rho = \lambda$,

$$\sigma_\rho = \frac{1}{\rho^2} \frac{\partial^2 \chi}{\partial \theta^2} + \frac{1}{\rho} \frac{\partial \chi}{\partial \rho} = 0, \quad \tau_{\rho\theta} = -\frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \frac{\partial \chi}{\partial \theta} \right) = 0$$

We write

$$\chi = \chi'_0 + \chi_0 + \chi_1 + \chi_2 + \dots + \chi_{2r} + \chi_{2r+1} + \chi_{2r+2} + \dots \tag{2}$$

where each term of the series satisfies the equation (1) separately, and has, in addition,

following properties:

χ'_0 gives the stresses at infinity. $\chi'_0 + \chi_0$ satisfies the condition on the edge of the hole and at infinity, but not on the straight edges, i.e., it is the solution for an infinite plate with a hole. χ_1 cancels the stresses due to χ_0 on the edges $y = \pm 1$ and satisfies the condition of the continuity of the normal strain to the flanges on the edges $y = \pm 1$, but introduces stresses on the edge of the hole. χ_2 cancels these, but again does not satisfy the boundary conditions on the straight edges. More generally, $\chi_{2r} + \chi_{2r+1}$ satisfies the boundary conditions on $y = \pm 1$, while $\chi_{2r+1} + \chi_{2r+2}$ gives zero stresses over $\rho = \lambda$.

Now we derive equations from which χ_{2r+1} and χ_{2r+2} may be calculated. The value of χ_{2r} will be assumed to be given in the form

$$\chi_{2r} = \sum_{n=0}^{\infty} \left\{ \frac{D_{2n+1}^{(r)}}{\rho^{2n+1}} + \frac{E_{2n+1}^{(r)}}{\rho^{2n-1}} \right\} \cos(2n+1)\theta \quad (3)$$

where $D_{2n+1}^{(r)}$ and $E_{2n+1}^{(r)}$ are coefficients, to be determined later. The stresses due to χ_{2r} are given as follows

$$\begin{aligned} \sigma_\rho &= \frac{1}{\rho} \frac{\partial \chi_{2r}}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \chi_{2r}}{\partial \theta^2} \\ &= -2 \sum_{n=0}^{\infty} \left\{ \frac{(n+1)(2n+1)D_{2n+1}^{(r)}}{\rho^{2n+3}} + \frac{n(2n+3)E_{2n+1}^{(r)}}{\rho^{2n+1}} \right\} \cos(2n+1)\theta \end{aligned} \quad (4)$$

$$\begin{aligned} \tau_{\rho\theta} &= -\frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \frac{\partial \chi_{2r}}{\partial \theta} \right) \\ &= -2 \sum_{n=0}^{\infty} \left\{ \frac{(n+1)(2n+1)D_{2n+1}^{(r)}}{\rho^{2n+3}} + \frac{n(2n+1)E_{2n+1}^{(r)}}{\rho^{2n+1}} \right\} \sin(2n+1)\theta \end{aligned} \quad (5)$$

$$\begin{aligned} \sigma_\theta &= \frac{\partial^2 \chi_{2r}}{\partial \rho^2} \\ &= 2 \sum_{n=0}^{\infty} \left\{ \frac{(n+1)(2n+1)D_{2n+1}^{(r)}}{\rho^{2n+3}} + \frac{n(2n-1)E_{2n+1}^{(r)}}{\rho^{2n+1}} \right\} \cos(2n+1)\theta \end{aligned} \quad (6)$$

The stresses relative to the Cartesian axes are given by the equations.

$$\sigma_x = 2 \sum_{n=0}^{\infty} \left[\frac{(n+1)\{(2n+1)D_{2n+1}^{(r)} - 2E_{2n+1}^{(r)}\}}{\rho^{2n+3}} + \frac{n(2n+1)E_{2n+1}^{(r)}}{\rho^{2n+1}} \right] \cos(2n+1)\theta \quad (7)$$

$$\sigma_y = -2 \sum_{n=0}^{\infty} \left[\frac{(n+1)\{(2n+1)D_{2n+1}^{(r)} + 2E_{2n+1}^{(r)}\}}{\rho^{2n+3}} + \frac{n(2n+1)E_{2n+1}^{(r)}}{\rho^{2n+1}} \right] \cos(2n+1)\theta \quad (8)$$

$$\tau_{xy} = -2 \sum_{n=0}^{\infty} \left[\frac{(n+1)(2n+1)D_{2n+1}^{(r)}}{\rho^{2n+3}} + \frac{n(2n+1)E_{2n+1}^{(r)}}{\rho^{2n+1}} \right] \sin(2n+1)\theta \quad (9)$$

We have now to construct χ_{2r+1} so that it produces on the edges of the strip, the stresses cancelling the value of σ_y in (8), and making the value of shearing stress to be $-\bar{\psi}(x)$ after addition of the value of τ_{xy} in (9), where $-\bar{\psi}(x)$ is to be determined from the condition of the continuity of the normal strain.

Let χ_{2r+1} be the biharmonic function which is resolved as follows:

$$\left. \begin{aligned} \chi_{2r+1} &= \chi' + \chi'' \\ \chi' &= \frac{4}{\pi} \int_0^\infty \frac{uycC - (c+us)S}{u^2 \Sigma'} \cos uxdu \int_0^\infty \phi(w) \cos uw dw \\ \chi'' &= \frac{4}{\pi} \int_0^\infty \frac{ysC - cS}{u \Sigma'} \cos uxdu \int_0^\infty [\psi(w) - \bar{\psi}(w)] \sin uw dw \end{aligned} \right\} \quad (10)$$

where

$$s = \sinh u, \quad S = \sinh uy, \quad c = \cosh u, \quad C = \cosh uy, \quad \Sigma' = \sinh 2u - 2u \quad (11)$$

$$\left. \begin{aligned} \phi(x) &= 2 \sum_{n=0}^\infty \left[\frac{(n+1) \{ (2n+1) D_{2n+1}^{(r)} + 2E_{2n+3}^{(r)} \}}{(1+x^2)^{(2n+3)/2}} + \frac{n(2n+1) E_{2n+1}^{(r)}}{(1+x^2)^{(2n+1)/2}} \right] \cos (2n+3)\theta \\ \psi(x) &= 2 \sum_{n=0}^\infty \left[\frac{(n+1)(2n+1) D_{2n+1}^{(r)}}{(1+x^2)^{(2n+3)/2}} + \frac{n(2n+1) E_{2n+1}^{(r)}}{(1+x^2)^{(2n+1)/2}} \right] \sin (2n+3)\theta \\ \bar{\psi}(x) &= \text{unknown function}, \quad \theta = \tan^{-1}x. \end{aligned} \right\} \quad (12)$$

In order to determine the value of $\bar{\psi}(x)$, it is necessary to evaluate the strain at the line of connection with flange. By considering the condition, $\sigma_y=0$ on $y=1$, the unit elongation in the x -direction is written by

$$\begin{aligned} (\epsilon_x)_{y=1} &= \frac{1}{E} (\sigma_x)_{y=1} = \frac{1}{E} \left(\frac{\partial^2 \chi_{2r}}{\partial y^2} + \frac{\partial^2 \chi'}{\partial y^2} + \frac{\partial^2 \chi''}{\partial y^2} \right)_{y=1} \\ &= \frac{2}{\pi E} \int_0^\infty \left[2 \frac{sc+u}{\Sigma'} \int_0^\infty \phi(w) \cos uw dw + \frac{4s^2}{\Sigma'} \int_0^\infty \{ \psi(w) \right. \\ &\quad \left. - \bar{\psi}(w) \} \sin uw dw + \int_0^\infty \bar{\phi}(w) \cos uw dw \right] \cos ux du \quad (13) \end{aligned}$$

where E =Young's modulus

$$\bar{\phi}(x) = 2 \sum_{n=0}^\infty \left[\frac{(n+1) \{ (2n+1) D_{2n+1}^{(r)} - 2E_{2n+3}^{(r)} \}}{(1+x^2)^{(2n+3)/2}} + \frac{n(2n+1) E_{2n+1}^{(r)}}{(1+x^2)^{(2n+1)/2}} \right] \cos (2n+3)\theta \quad (14)$$

On the other hand, the unit elongation of the flange on the line of connection, $\bar{\epsilon}_x$ is given by the equation in such a form as

$$\bar{\epsilon}_x = \frac{2t_w}{\pi E t_f} \int_0^\infty g(u) \cos uxdu \int_0^\infty \bar{\psi}(w) \sin uw dw \quad (15)$$

where $g(u)$ is a function of u only and depends on the boundary conditions in the flanges.

The value of $g(u)$ is given at 2-3 for typical cases.

Equating (13) to (15), $\bar{\psi}(x)$ is given in the form

$$\left. \begin{aligned} \int_0^\infty \bar{\psi}(w) \sin uw dw &= \frac{1}{e^{2u} R} \left[2(sc+u) \int_0^\infty \phi(w) \cos uw dw \right. \\ &\quad \left. + 4s^2 \int_0^\infty \psi(w) \sin uw dw + \Sigma' \int_0^\infty \bar{\phi}(w) \cos uw dw \right] \end{aligned} \right\} \quad (16)$$

where

$$R = e^{-2u} \left[t_w g(u) \Sigma' + 4s^2 \right]$$

Substituting (16) into (10), and considering

$$\int_0^\infty \frac{\cos(2n+3)\theta \cos uw}{(1+w^2)^{(2n+3)/2}} dw = \int_0^\infty \frac{\sin(2n+3)\theta \sin uw}{(1+w^2)^{(2n+3)/2}} dw = \frac{\pi e^{-u} u^{2n+2}}{2(2n+2)!}$$

$$\int_0^\infty \frac{\cos(2n+3)\theta \cos uw}{(1+w^2)^{(2n+1)/2}} dw = \int_0^\infty \frac{\sin(2n+3)\theta \sin uw}{(1+w^2)^{(2n+1)/2}} dw = \frac{\pi e^{-u} u^{2n}(2u-2n-1)}{2(2n+1)!}$$

$$yC \cos ux = \frac{1}{2} \left[\rho \cos \theta + \sum_{n=0}^\infty \left\{ \frac{u^{2n} \rho^{2n+1}}{(2n)!} + \frac{u^{2n+2} \rho^{2n+3}}{(2n+2)!} \right\} \cos(2n+1)\theta \right]$$

$$S \cos ux = \sum_{n=0}^\infty \frac{(u\rho)^{2n+1}}{(2n+1)!} \cos(2n+1)\theta$$

we may now express χ_{2r+1} in the form

$$\chi_{2r+1} = \sum_{n=0}^\infty [L_{2n+1}^{(r)} + M_{2n+1}^{(r)} \rho^2] \rho^{2n+1} \cos(2n+1)\theta \tag{17}^*$$

$$\left. \begin{aligned} L_{2n+1}^{(r)} &= \sum_{p=0}^\infty \{ \kappa_{2p+1}^{2n+1} D_{2p+1}^{(r)} + \mu_{2p+1}^{2n+1} E_{2p+1}^{(r)} \} & n \geq 1 \\ M_{2n+1}^{(r)} &= \sum_{p=0}^\infty \{ \nu_{2p+1}^{2n+1} D_{2p+1}^{(r)} + \omega_{2p+1}^{2n+1} E_{2p+1}^{(r)} \} & n \geq 0 \end{aligned} \right\} \tag{18}$$

where

$$\left. \begin{aligned} \kappa_{2p+1}^{2n+1} &= \frac{1}{(2n+1)!(2p)!} [2nI'_{2n+2p+1} - 2I'_{2n+2p+2} - J'_{2n+2p+1} \\ &\quad - (2n+1)K'_{2n+2p+1} + 2H'_{2n+2p+2}] \\ \mu_{2p+1}^{2n+1} &= \frac{1}{(2n+1)!(2p-1)!} [4(n+p)I'_{2n+2p} - 4I'_{2n+2p+1} - (4np+1)I'_{2n+2p-1} \\ &\quad + 2(n+p)J'_{2n+2p-1} + (2p+1)(2n+1)K'_{2n+2p-1} - 2(2n+2p+2)H'_{2n+2p} \\ &\quad + 4F'_{2n+2p+1}] \\ \nu_{2p+1}^{2n+1} &= \frac{1}{(2n+2)!(2p)!} [I'_{2n+2p+3} - K'_{2n+2p+3}] \\ \omega_{2p+1}^{2n+1} &= \frac{1}{(2n+2)!(2p-1)!} [2I'_{2n+2p+2} - 2pI'_{2n+2p+1} + J'_{2n+2p+1} + (2p+1)K'_{2n+2p+1} \\ &\quad - 2H'_{2n+2p+2}] \end{aligned} \right\} \tag{19}$$

$$\left. \begin{aligned} I'_s &= \int_0^\infty \frac{u^s}{\Sigma'} du, & J'_s &= \int_0^\infty \frac{u^s}{\Sigma'} e^{-2u} du \\ F'_s &= \int_0^\infty \frac{(1+e^{-2u})^2}{R\Sigma'} u^s du, & H'_s &= \int_0^\infty \frac{(1-e^{-4u})}{R\Sigma'} u^s du, & K'_s &= \int_0^\infty \frac{(1-e^{-2u})^2}{R\Sigma'} u^s du \end{aligned} \right\} \tag{20}$$

F'_s, H'_s and K'_s represent the effects of the flange and vanish when the perforated strip has no flange.

The coefficients in χ_{2r+1} are thus determined in terms of those of χ_{2r} . To com-

* As the term containing $L_1^{(r)}$ is without effect on the stresses, it is trivial and will be supposed omitted.

plete the cycle, we have now to determine the coefficients in χ_{2r+2} in terms of those of χ_{2r+1} . The stresses due to χ_{2r+1} are given immediately by the differentiation of (17) as

$$\left. \begin{aligned} \sigma_\rho &= -2 \sum_{n=0}^{\infty} [n(2n+1)L_{2n+1}^{(r)} + (n+1)(2n-1)M_{2n+1}^{(r)}\rho^2] \rho^{2n-1} \cos(2n+1)\theta \\ \tau_{\rho\theta} &= 2 \sum_{n=0}^{\infty} [n(2n+1)L_{2n+1}^{(r)} + (n+1)(2n+1)M_{2n+1}^{(r)}\rho^2] \rho^{2n-1} \sin(2n+1)\theta \\ \sigma_\theta &= 2 \sum_{n=0}^{\infty} [n(2n+1)L_{2n+1}^{(r)} + (n+1)(2n+3)M_{2n+1}^{(r)}\rho^2] \rho^{2n-1} \cos(2n+1)\theta \end{aligned} \right\} \quad (21)$$

The first two of these must be cancelled at the edge of the hole by the stresses due to χ_{2r+2} . If we express χ_{2r+2} in such a form as χ_{2r} ;

$$\chi_{2r+2} = \sum_{n=0}^{\infty} \left\{ \frac{D_{2n+1}^{(r+1)}}{\rho^{2n+1}} + \frac{E_{2n+1}^{(r+1)}}{\rho^{2n-1}} \right\} \cos(2n+1)\theta \quad (22)$$

the corresponding stresses will be given by the equations (4) and (5), replacing the suffix (r) in the coefficients by ($r+1$). Putting $\rho=\lambda$ and equating the coefficients the negative of those in (21), we obtain the following equations:

$$\left. \begin{aligned} D_{2n+1}^{(r+1)} &= [2nL_{2n+1}^{(r)} + (2n+1)M_{2n+1}^{(r)}\lambda^2] \lambda^{4n+2} \\ E_{2n+1}^{(r+1)} &= -[(2n+1)L_{2n+1}^{(r)} + 2(n+1)M_{2n+1}^{(r)}\lambda^2] \lambda^{4n} \end{aligned} \right\} \quad (23)$$

Substituting (18) into (23),

$$\left. \begin{aligned} D_{2n+1}^{(r+1)} &= \sum_{p=0}^{\infty} \{ {}^{2n+1}s_{2p+1} D_{2p+1}^{(r)} + {}^{2n+1}t_{2p+1} E_{2p+1}^{(r)} \} \\ E_{2n+1}^{(r+1)} &= \sum_{p=0}^{\infty} \{ {}^{2n+1}u_{2p+1} D_{2p+1}^{(r)} + {}^{2n+1}v_{2p+1} E_{2p+1}^{(r)} \} \\ {}^{2n+1}s_{2p+1} &= \lambda^{4n+2} \{ 2n^{2n+1} \kappa_{2p+1} + (2n+1) \lambda^{2 \cdot 2n+1} \nu_{2p+1} \} \\ {}^{2n+1}t_{2p+1} &= \lambda^{4n+2} \{ 2n^{2n+1} \mu_{2p+1} + (2n+1) \lambda^{2 \cdot 2n+1} \omega_{2p+1} \} \\ {}^{2n+1}u_{2p+1} &= -\lambda^{4n} \{ (2n+1)^{2n+1} \kappa_{2p+1} + 2(n+1) \lambda^{2 \cdot 2n+1} \nu_{2p+1} \} \\ {}^{2n+1}v_{2p+1} &= -\lambda^{4n} \{ (2n+1)^{2n+1} \mu_{2p+1} + 2(n+1) \lambda^{2 \cdot 2n+1} \omega_{2p+1} \} \end{aligned} \right\} \quad (24)$$

By using (24), the coefficients in χ_{2r+2} are determined in terms of those of χ_{2r} .

2-2 Determination of the Stresses

To determine the stress function of the perforated strip under pure bending, we start with the stress function χ'_0 , of the unperforated strip under pure bending.

$$\chi'_0 = \frac{T}{24} \rho^3 (\cos 3\theta + 3 \cos \theta) \quad (25)$$

satisfies the definition. And the stress function χ_0 , which produces on the edge of the hole stresses cancelling the values of σ_ρ and $\tau_{\rho\theta}$ due to χ'_0 , are obtained by replacing the coefficients in (3) by the following values:

$$\left. \begin{aligned} D_1^{(0)} &= \frac{T}{8} \lambda^4, \quad D_3^{(0)} = \frac{T}{12} \lambda^6, \quad E_3^{(0)} = -\frac{T}{8} \lambda^4 \\ \text{and all the other coefficients are zero.} \end{aligned} \right\} \quad (26)$$

Considering (3), (17) and (25), the final value of χ is given by

$$\begin{aligned} \chi = \frac{T}{24} \rho^3 (\cos 3\theta + 3 \cos \theta) + \frac{T}{24} \lambda^4 \sum_{n=0}^{\infty} \left\{ \frac{d_{2n+1}}{\rho^{2n+1}} + \frac{e_{2n+1}}{\rho^{2n-1}} + l_{2n+1} \rho^{2n+1} \right. \\ \left. + m_{2n+1} \rho^{2n+3} \right\} \cos (2n+1)\theta \end{aligned} \quad (27)$$

where

$$\left. \begin{aligned} d_{2n+1} &= \frac{24}{T \lambda^4} \sum_r D_{2n+1}^{(r)}, \quad e_{2n+1} = \frac{24}{T \lambda^4} \sum_r E_{2n+1}^{(r)} \\ l_{2n+1} &= \frac{24}{T \lambda^4} \sum_r L_{2n+1}^{(r)}, \quad m_{2n+1} = \frac{24}{T \lambda^4} \sum_r M_{2n+1}^{(r)} \end{aligned} \right\} \quad (28)$$

The coefficients d_{2n+1} and e_{2n+1} in (28) are obtained by using (24) and starting with (26). The coefficients l_{2n+1} and m_{2n+1} are given by equation

$$\left. \begin{aligned} l_{2n+1} &= \sum_{p=0}^{\infty} \left\{ {}^{2n+1} \kappa_{2p+1} d_{2p+1} + {}^{2n+1} \mu_{2p+1} e_{2p+1} \right\} \\ m_{2n+1} &= \sum_{p=0}^{\infty} \left\{ {}^{2n+1} \nu_{2p+1} d_{2p+1} + {}^{2n+1} \omega_{2p+1} e_{2p+1} \right\} \end{aligned} \right\} \quad (29)$$

The equation (29) being obtained by (18) and (28).

The stresses due to χ are

$$\left. \begin{aligned} \sigma_\rho &= \frac{T}{4} \rho (\cos \theta - \cos 3\theta) - \frac{T}{12} \lambda^4 \sum_{n=0}^{\infty} \left\{ \frac{(n+1)(2n+1)d_{2n+1}}{\rho^{2n+3}} + \frac{n(2n+3)e_{2n+1}}{\rho^{2n+1}} \right. \\ &\quad \left. + n(2n+1)l_{2n+1}\rho^{2n-1} + (n+1)(2n-1)m_{2n+1}\rho^{2n+1} \right\} \cos (2n+1)\theta \\ \tau_{\rho\theta} &= \frac{T}{4} \rho (\sin 3\theta + \sin \theta) - \frac{T}{12} \lambda^4 \sum_{n=0}^{\infty} \left\{ \frac{(n+1)(2n+1)d_{2n+1}}{\rho^{2n+3}} + \frac{n(2n+1)e_{2n+1}}{\rho^{2n+1}} \right. \\ &\quad \left. - n(2n+1)l_{2n+1}\rho^{2n-1} - (n+1)(2n+1)m_{2n+1}\rho^{2n+1} \right\} \sin (2n+1)\theta \\ \sigma_\theta &= \frac{T}{4} \rho (\cos 3\theta + 3 \cos \theta) + \frac{T}{12} \lambda^4 \sum_{n=0}^{\infty} \left\{ \frac{(n+1)(2n+1)d_{2n+1}}{\rho^{2n+3}} + \frac{n(2n-1)e_{2n+1}}{\rho^{2n+1}} \right. \\ &\quad \left. + n(2n+1)l_{2n+1}\rho^{2n-1} + (n+1)(2n+3)m_{2n+1}\rho^{2n+1} \right\} \cos (2n+1)\theta \end{aligned} \right\} \quad (30)$$

The circumferential stress on the edge of the hole is obtained, by putting $\rho = \lambda$ in the third of the equations (30),

$$\left. \sigma_\theta = \frac{T}{12} \sum_{n=0}^{\infty} S_{2n+1} \cos (2n+1)\theta \right\}$$

where

$$\left. \begin{aligned} S_1 &= \lambda\{(9+d_1)+3m_1\lambda^4\} \\ S_3 &= 6d_3/\lambda+(3+e_3)\lambda+3l_3\lambda^5+10m_3\lambda^7 \\ S_5 &= 15d_5/\lambda^3+6e_5/\lambda+10l_5\lambda^7+21m_5\lambda^9 \\ S_7 &= 28d_7/\lambda^5+15e_7/\lambda^3+21l_7\lambda^9+36m_7\lambda^{11} \\ S_9 &= 45d_9/\lambda^7+28e_9/\lambda^5+36l_9\lambda^{11}+55m_9\lambda^{13} \\ &..... \end{aligned} \right\} \quad (31)$$

The value of σ_θ is the maximum at $\theta=0^\circ$ and zero at $\theta=90^\circ$.

2-3 The values of $g(u)$ and R

We will determine $g(u)$ and R for the three typical cases as shown in Fig. 2. It is seen that the boundary conditions in the flange are same as in the previous tension problem¹⁾. Therefore the values of $g(u)$ are the same as these obtained in tension problem.

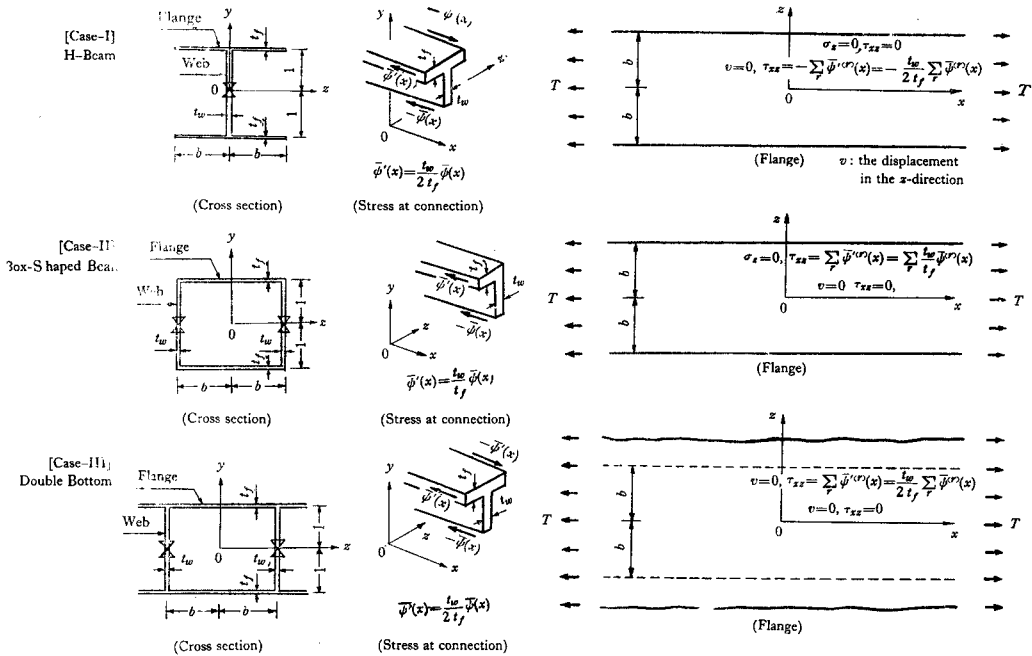


Fig. 2. Typical structures and these boundary condition.

The values of R are easily obtained by substituting the values of $g(u)$ into the equation (16). The values of $g(u)$ and R are written as follows.

[Case-I] H-Beam

$$\left. \begin{aligned} g(u) &= \frac{1}{(1+j)\bar{\Sigma}} [(u^2b^2+j\bar{s}^2)(1+\nu)+2(j+\bar{c}^2)] \\ R &= \frac{t_w}{2t_f}\frac{\bar{\Sigma}'}{\bar{\Sigma}} e^{-2u} \left[(1+\nu)^2u^2b^2 + \frac{(1+\nu)(3-\nu)}{4} (e^{2ub} + e^{-2ub}) + \frac{5-2\nu+\nu^2}{2} \right] + (1-e^{-2u})^2 \end{aligned} \right\} \quad (32)$$

where

$$\left. \begin{aligned} \nu &= \text{Poisson's ratio, } j = (1-\nu)/(1+\nu) \\ \bar{\Sigma} &= \sinh 2ub + 2ub, \quad \bar{s} = \sinh ub, \quad \bar{c} = \cosh ub \end{aligned} \right\} \quad (33)$$

[Case-II] Box-Shaped Beam

$$g(u) = 4 \frac{\bar{c}^2}{\bar{\Sigma}}, \quad R = \frac{t_w}{t_f} \frac{\Sigma'}{\bar{\Sigma}} e^{-2u(1-b)} (1+e^{-2ub})^2 + (1-e^{-2u})^2 \quad (34)$$

[Case-III] Double Bottom

$$\left. \begin{aligned} g(u) &= \frac{1}{2(1+j)} \frac{(3-\nu)\bar{s}\bar{c} - (1+\nu)ub}{\bar{s}^2} \\ R &= \frac{t_w e^{-2u} \Sigma'}{2t_f (1-e^{-2ub})^2} \left[\frac{(1+\nu)(3-\nu)}{2} (1-e^{-4ub}) - 2(1+\nu)^2 ub e^{-2ub} \right] + (1-e^{-2u})^2 \end{aligned} \right\} \quad (35)$$

3. The numerical values of the stresses

The numerical calculation has been carried out for $\nu=0.3$. As an example, the stress distribution around the hole for the H-beam of $b=0.5$ and $t_f/t_w=1$ are shown graphically in Fig. 3, and are compared with those in a perforated strip.

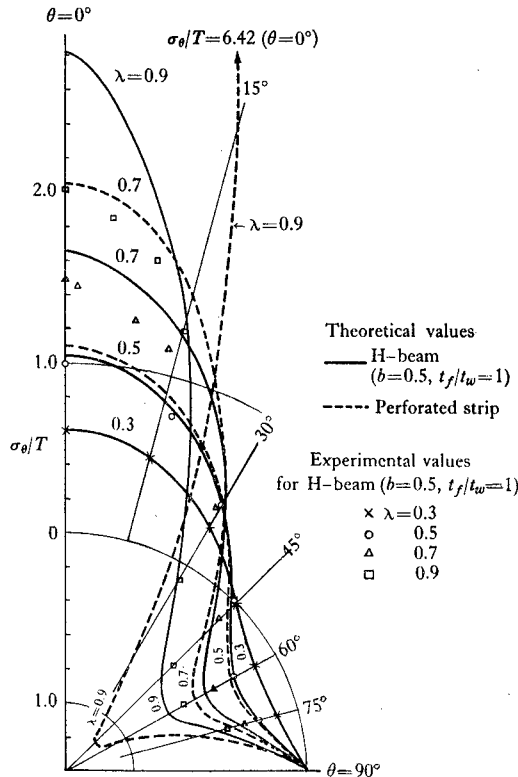


Fig. 3.

The maximum stresses around the hole $\sigma_{\theta\max}$ for three typical types of beams, such as H-beam, box-shaped beam and double bottom, in which the flanges of the same size are attached to one web, are plotted in Fig. 4 versus the value of λ . Solid lines shows

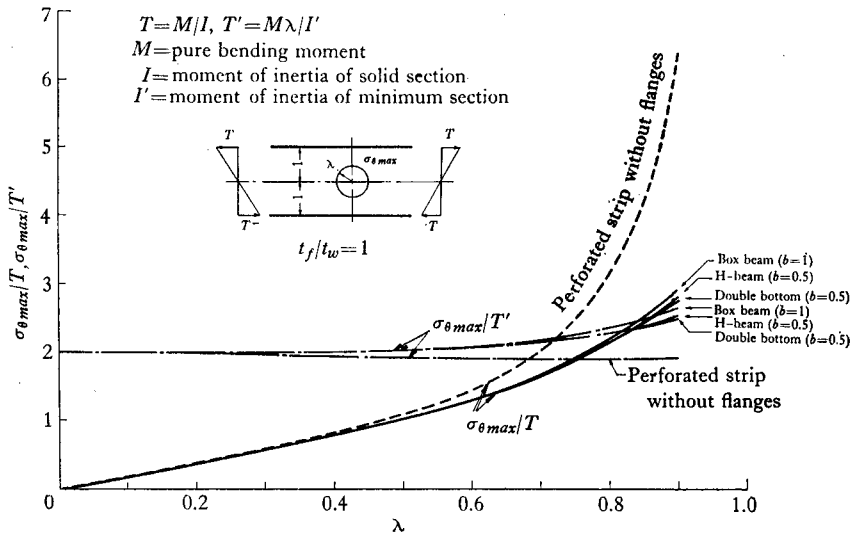


Fig. 4.

the values of $\sigma_{\theta\max}/T$. From these, it is seen that the effect of the differences between types of beams on the value of $\sigma_{\theta\max}$ is quite minor. Comparing these σ_{θ}/T -values with the value for perforated strip (broken line), it is found that the presence of flange makes the value of $\sigma_{\theta\max}$ decrease, and this effect is remarkable when hole is very large. On the other hand, from comparison between the chain lines in Fig. 4 which show the value

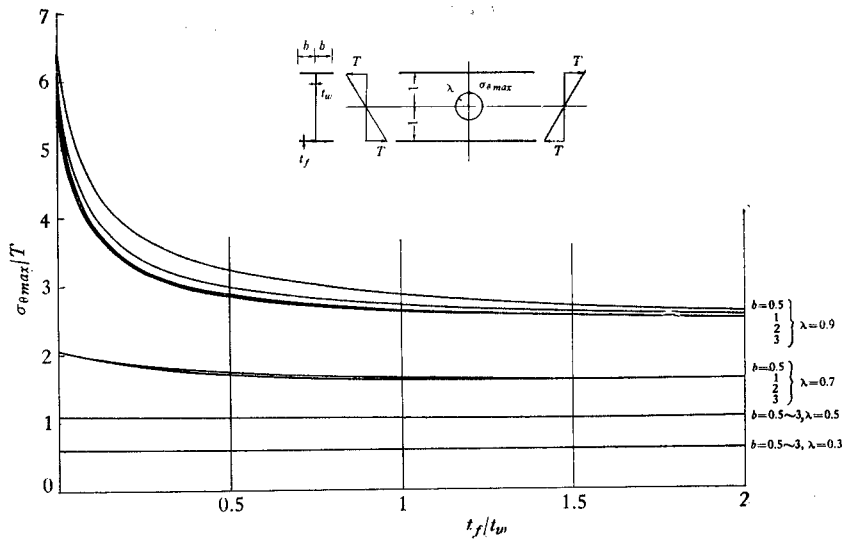


Fig. 5.

of σ_θ/T' (T' =the nominal bending stress at the edge of the hole in the minimum section), it may be said that the effect of the flange on the value of $\sigma_{\theta\max}$ is smaller than the effect of the flange on the nominal bending stress at the edge of the hole in the minimum section.

To show the effect of flange size on the maximum stress around the hole, the values of $\sigma_{\theta\max}/T$ versus the values of t_f/t_w for H-beam are shown in Fig. 5. The values of $\sigma_{\theta\max}/T$ decrease as the values of t_f/t_w increase. The rate of decrease is large in large values of λ and small values of t_f/t_w , and zero in small values of λ or large values of t_f/t_w .

The effect of b on $\sigma_{\theta\max}/T$, being similar to the effect of t_f/t_w mentioned above, is large in large values of λ and small values of b . The values of $\sigma_{\theta\max}/T$ rapidly decrease to the asymptotic value as the values of b increase.

4. Experiment

We made tests with welded steel model of H section. The model is shown in Fig. 6. For the diameter of the hole 30 mm was originally provided; successive machining operations enlarged it in steps of 20 mm to 90 mm. As the depth of web, the distance center to center of flange was used. The Young's modulus and Poisson's ratio of steel were 2.08×10^4 Kg/mm² and 0.3, respectively. The strains were measured with electric resistance strain gage with the length of 2 mm.

The experimental values of σ_θ/T were plotted in Fig. 3, where T is the nominal bending stress at the mid plane of the flange for the uncut beam. The diagrams offer the close agreement between theory and experiments when hole is small, while when hole

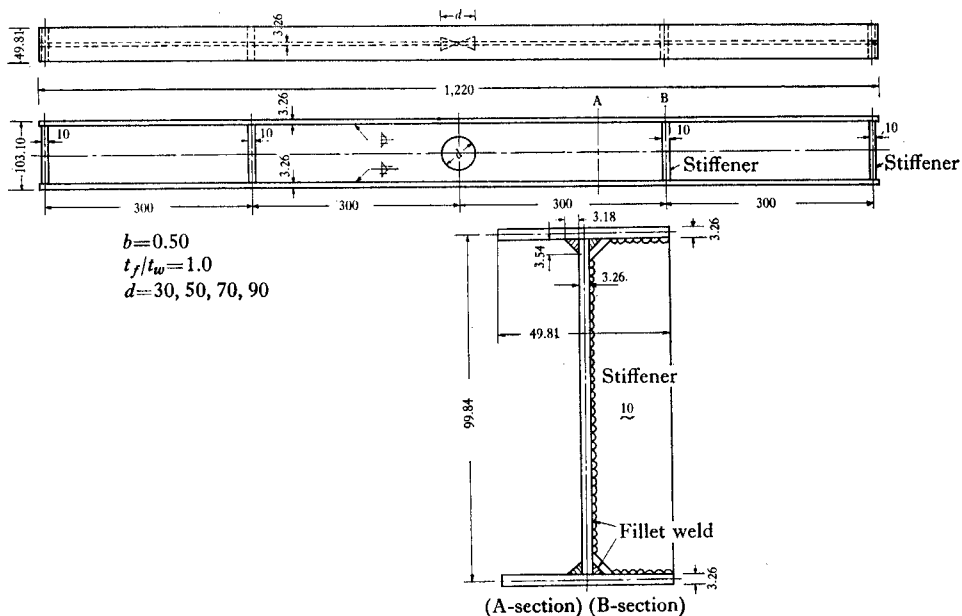


Fig. 6.

is large the experimental values are somewhat smaller than the theoretical, probably due to the effect of weld area.

5. Conclusions

The formulae for the stresses around the hole in the web of the beam subjected to pure bending were theoretically derived for the typical three cases. By the results of numerical calculation, the effects of the flange on the stresses around the hole were illustrated. Moreover, the theoretical results were compared with the experimental.

The conclusions drawn are such as follows:

1) The effect of the flange on the stresses around the hole is remarkable in very large hole and the stresses around the hole are made to decrease by the presence of flange, but the effect of the flange on the maximum stress around the hole is smaller than the effect of the flange on the nominal bending stress at the edge of the hole in the minimum section. The rate of decrease of maximum stress around the hole for the increase of flange size is large in the small size of flange and zero in the large.

2) When the flanges of the same size are attached to one web, the maximum stresses around the hole for three typical types of beams are close.

3) The experimental values agree with the theoretical for small hole, but the experimental values are smaller than the theoretical for the large hole, probably due to the effect of the weld area.

References

- 1) Y. Okamura, C. Yamabe and Y. Fukumoto; Bull. of Univ. of Osaka Pref., A **11**, no. 1, p. 99 (1962).
- 2) M. Frocht and M. Leven; Trans. ASME, **73**, p. 107 (1951).