A Note on Tax Evasion : Tax Evasion of Multi-product Monopolist & Tax Evasion as a Built-in Stabilizer

著者	Watanabe Shigeru
引用	Journal of economics, business and law. 2005, 7, p.87-95
URL	http://doi.org/10.24729/00000891

A Note on Tax Evasion*
Tax Evasion of Multi-product Monopolist
&
Tax Evasion as a Built-in Stabilizer

Shigeru Watanabe**

1. Introduction

In the next section of this note the relationship between multi-product monopolist[1] and tax evasion[2] will be examined. Using the model developed by Lambertini (2003, 2004) and Ping Lin (2004) a simple model of tax evasion by the multi-product monopolist will be analyzed. From the analysis following main results will be derived; in the ordinary case where the monopoly produces only one product raising the tax rate will increase the output level of the product and raising the penalty rate will decrease that output level, on the other hand in the multi-product case the effect of raising the tax rate or that of raising the penalty rate can not be determined in general and will depend on the condition whether the products offered by the monopolist are substitutes or complements. In section 3 of this note the tax evasion as a built-in stabilizer will be examined. In the last section, concluding remarks will be given.

2. A Simple model of Tax Evasion by Multi-product Monopolist

The prices for the products x_i (i = 1,...,n) offered by the multi-product monopoly firm are given by

$$P_i = \alpha - x_i - \gamma \sum_{j \neq i} x_j \quad , \quad i = 1, \cdots , n \tag{1}$$

* I thank professor Y. Tomita for helpful comments.
** Professor at College of Economics, Osaka Prefecture University , 1 – 1, Gakuen-cho, Sakai, Osaka, 599-8531,Japan

The products are substitute if $\gamma \in (0, 1)$ and complements if $\gamma \in (-1, 0)$. Total costs C of the firm can be denoted by

$$c = \sum_{i=1}^{n} c_i x_i + \theta n M , \tag{2}$$

where θ is the scope economies parameter in production with $\theta \in (0, 1)$ for n > 1 and $\theta = 1$ for n = 1, the marginal cost of production for each product is denoted by c_i which is assumed to be given since in this note the level of process R&D is not taken into consideration to focus on the tax evasion, and M is the fixed cost of introducing a product.

Hence the profit π is given by

$$\pi = \sum_{i=1}^{n} \left[\alpha - x_i - \gamma \sum_{j \neq i} x_j - c_i \right] x_i - n \theta M \tag{3}$$

Therefore, the expected profit $E\pi$ of the multi-product monopolist can be denoted by

$$\begin{aligned} E\pi = & (1-t)\{(\alpha - x_1 - \gamma x_2)x_1 + (\alpha - x_2 - \gamma x_1)x_2 \\ & - c_1 x_1 - c_2 x_2 - 2\theta M \} \\ & + (1 - q(\delta)F)t\delta(c_1 x_1 + c_2 x_2 + 2\theta M), \end{aligned} \tag{4}$$

where t is the tax rate, F is the penalty rate with respect to tax evasion $q(\delta)$ is the probability of detection, δ is the rate of cost-overstatement and to simplify the analysis n is assumed to be 2.

Maximizing $E\pi$ with respect to δ, x_1 and x_2 yields the first order conditions.

$$\begin{aligned} \frac{\partial E\pi}{\partial \delta} & = t(1 - 2\delta F)(c_1 x_1 + c_2 x_2 + 2\theta M) \\ & = 0, \end{aligned} \tag{5}$$

$$\frac{\partial E\pi}{\partial x_1} = (1-t)(\alpha - 2x_1 - 2\gamma x_2 - c_1) + (1 - q(\delta)F)t\delta c_1 = 0, \tag{6}$$

$$\frac{\partial E\pi}{\partial x_2} = (1-t)(\alpha - 2x_2 - 2\gamma x_1 - c_2) + (1 - q(\delta)F)t\delta c_2 = 0. \tag{7}$$

Second order conditions are satisfied;

$$\frac{\partial^2 E\pi}{\partial \delta^2} = -2t(c_1x_1 + c_2x_2 + 2\theta M)F < 0, \tag{8}$$

$$\begin{vmatrix} \dfrac{\partial^2 E\pi}{\partial \delta^2} & \dfrac{\partial^2 E\pi}{\partial \delta \partial x_1} \\ \dfrac{\partial^2 E\pi}{\partial x_1 \partial \delta} & \dfrac{\partial^2 E\pi}{\partial x_1^2} \end{vmatrix} = 4(1-t)t(c_1x_1 + c_2x_2 + 2\theta M)F > 0, \tag{9}$$

$$\begin{vmatrix} \dfrac{\partial^2 E\pi}{\partial \delta^2} & \dfrac{\partial^2 E\pi}{\partial \delta \partial x_1} & \dfrac{\partial^2 E\pi}{\partial \delta \partial x_2} \\ \dfrac{\partial^2 E\pi}{\partial x_1 \partial \delta} & \dfrac{\partial^2 E\pi}{\partial x_1^2} & \dfrac{\partial^2 E\pi}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 E\pi}{\partial x_2 \partial \delta} & \dfrac{\partial^2 E\pi}{\partial x_2 \partial x_1} & \dfrac{\partial^2 E\pi}{\partial x_2^2} \end{vmatrix} = -8t(1-t)^2(1-\gamma^2)(c_1x_1 + c_2x_2 + 2\theta M)F < 0. \tag{10}$$

where $\gamma^2 < 1$ since $\gamma \in (0, 1)$ if the products are substitutes and $\gamma \in (-1, 0)$ if the products are complements.

From the first order conditions the following results can be obtained;

$$\delta^* = \frac{1}{2F}, \tag{11}$$

$$x_1{}^* = \frac{a - c_1 + \dfrac{c_1 t}{4(1-t)F} - \gamma\left\{a - c_2 + \dfrac{tc_2}{4(1-t)F}\right\}}{2\left(1-\gamma^2\right)}, \tag{12}$$

$$x_2{}^* = \frac{a - c_2 + \dfrac{c_2 t}{4(1-t)F} - \gamma\left\{a - c_1 + \dfrac{tc_1}{4(1-t)F}\right\}}{2\left(1-\gamma^2\right)}. \tag{13}$$

Differentiating (11) , (12) , (13) with respect to F yields

$$\frac{\partial \delta^*}{\partial F} < 0, \tag{14}$$

$$\frac{\partial x_1{}^*}{\partial F} = \frac{-t(c_1 - \gamma c_2)}{8(1-t)\left(1-\gamma^2\right)F^2} \begin{matrix} > \\ = \\ < \end{matrix} 0, \tag{15}$$

according to $\dfrac{c_1}{c_2} \begin{matrix} < \\ = \\ > \end{matrix} \gamma$.

$$\frac{\partial x_2{}^*}{\partial F} = \frac{-t(c_2 - \gamma c_1)}{8(1-t)\left(1-\gamma^2\right)F^2} \begin{matrix} > \\ = \\ < \end{matrix} 0, \tag{16}$$

according to $\dfrac{c_2}{c_1} \begin{matrix} < \\ = \\ > \end{matrix} \gamma$.

Therefore, if the products are compliments i.e., $\gamma \in (-1, 0)$ or the monopolist is not the multi-product firm i.e., $\gamma = 0$ then raising the penalty rate will decrease the output level of the products. On the other hand , if the products are substitutes $\gamma \in (0, 1)$ and $c_1 / c_2 < \gamma$, then raising the penalty rate will increase the output level of the product x_1 , and if the products are substitutes and $c_2 / c_1 < \gamma$, then raising the penalty rate will increase the output level of the product x_2 .

Next, differentiating (11), (12) and (13) with respect to t yields

$$\frac{\partial \delta^*}{\partial t} = 0, \tag{17}$$

$$\frac{\partial x_1^*}{\partial t} = \frac{(c_1 - \gamma c_2)F}{8(1-\gamma^2)(1-t)^2 F^2} \tag{18}$$

$$\frac{\partial x_2^*}{\partial t} = \frac{(c_2 - \gamma c_1)F}{8(1-\gamma^2)(1-t)^2 F^2} \tag{19}$$

Hence, $$\frac{\partial x_1^*}{\partial t} \gtreqless 0 , \tag{20}$$

according to $$\frac{c_1}{c_2} \gtreqless \gamma .$$

$$\frac{\partial x_2^*}{\partial t} \gtreqless 0 , \tag{21}$$

according to $$\frac{c_2}{c_1} \gtreqless \gamma .$$

Therefore, if the products are compliments i.e., $\gamma \in (-1, 0)$ or the monopolist is not the multi-product firm i.e., $\gamma = 0$ then raising the tax rate will increase the output level of the products. On the other hand , if the products are substitutes $\gamma \in (0, 1)$ and $c_1 / c_2 < \gamma$, then raising the tax rate will decrease the output level of the product x_1, and if the products are substitutes and $c_2 / c_1 < \gamma$, then raising the tax rate will decrease the output level of the product x_2.

In addition to the above results the following results can also be derived straightforwardly;

$$\frac{\partial[x_1*+x_2*]}{\partial F}=\frac{-t(1-\gamma)(c_1+c_2)}{8(1-t)(1-\gamma^2)F^2}<0, \tag{22}$$

$$\frac{\partial[x_1*+x_2*]}{\partial t}=\frac{(1-\gamma)(c_1+c_2)F}{8(1-\gamma^2)(1-t)^2F^2}>0. \tag{23}$$

Therefore with respect to the sum of the output levels, the effect of raising the penalty rate or that of raising the tax rate on the sum of the products does not depend on the condition whether the products are substitutes or compliments. Raising the tax rate or decreasing the penalty rate will increase the sum of the output levels.

3. Tax evasion as a Built-in Stabilizer

In this section using a model as simple as possible, the effect of an increase in investment on national income in presence of tax evasion will be compared with that effect in absence of tax evasion.

Consumption C in presence of tax evasion is denoted by

$$C=c_0+c_1(1-t)\mu(t)Y+c_2(1-\mu(t))Y, \tag{24}$$

where c_0 is constant c_1 is the marginal propensity to consume from the reported income c_2 is the marginal propensity to consume from the unreported income t is the tax rate $\mu(t)$ is the rate at which income is reported to the tax authority and Y is national income.

Hence the effect of an increase in investment on national income in presence of tax evasion is shown as

$$\left.\frac{\partial Y}{\partial I}\right|_{0<\mu(t)<1}=\frac{1}{1-c_1(1-t)\mu(t)-c_2(1-\mu(t))} \quad . \tag{25}$$

On the other hand the effect in absence of the tax evasion is shown as

$$\left.\frac{\partial Y}{\partial I}\right|_{\mu(t)=1} = \frac{1}{1-c_1(1-t)} \quad . \tag{26}$$

Therefore the following relation can be derived straightforwardly

$$\left.\frac{\partial Y}{\partial I}\right|_{0<\mu(t)<1} \quad \begin{matrix} < \\ = \\ > \end{matrix} \quad \left.\frac{\partial Y}{\partial I}\right|_{\mu(t)=1} \tag{27}$$

according to $\frac{c_1}{c_2} \begin{matrix} > \\ = \\ < \end{matrix} \frac{1}{1-t}.$

Therefore, in a special case where $c_1 = c_2$, $1 < 1/(1-t)$, the effect of an increase in investment on national income in absence of tax evasion is milder than the effect in presence of tax evasion. Therefore the tax evasion can not be regarded as a built—in stabilizer. However, in general whether the tax evasion is a built—in stabilizer or not depends on the condition shown by (27). The higher the marginal propensity to consume with respect to the reported income or the lower the marginal propensity to consume with respect to the unreported income, or the lower the tax rate, the higher the probability that the tax evasion has an aspect as a built—in stabilizer, though the tax evasion can not be accepted from the point of equity.

4. Concluding Remarks

In this note using the model developed by Lambertini (2003, 2004) and Ping Lin (2004) the relationship between multi-product monopoly and tax evasion has been examined. Following main results have been derived; in the multi-product case the effect of raising the tax rate or that of raising the penalty rate can not be determined in general and will depend on the condition whether the products offered by the monopolist are substitutes or complements, though in the ordinary case where the monopoly produces only one product raising the tax rate will always increase the output level of the product and raising the penalty rate will always decrease that output level. If the products are compliments raising the tax rate or decreasing the penalty rate will increase the output level of the products. On the other hand, if the products are substitutes and the condition with respect to the marginal costs denoted above is satisfied raising the tax rate or decreasing the penalty rate will not increase the output levels of the products but will decrease them. With respect to the sum of the output levels the effect of raising the penalty rate or that of raising the tax rate on the sum of the products does not depend on the condition whether the products are substitutes or compliments. Raising the tax rate or decreasing the penalty rate will increase the sum of the output levels.

Concerning the built－in stabilizer ,in general, the higher the marginal propensity to consume with respect to the reported income or the lower the marginal propensity to consume with respect to the unreported income, or the lower the tax rate, the higher the probability that the tax evasion has an aspect as a built－in stabilizer, though the tax evasion can not be accepted from the point of equity.

1 See Lambertini, L (2003, 2004) and Ping Lin (2004)

2 See M.G.Allingham and A.Sandmo (1972), A.T.Peacock and Show,G.K.(1982), G. Laszlo (2004)and Watanabe (1986,1987, 1989)

3 See, for instance, Musgrave (1959) .

FERERENCES

M.G.Allingham and A. Sandmo, "Income Tax Evasion : A Theoretical Analysis", ***Journal of Public Economics,*** 1, 1972.

L. Lambertini, L, "The monopolist's optimal R & D portfolio" ***Oxford Economic Paper,*** 2003.

L. Lambertini, L, "Process and product R & D by a multiproduct monopolist: : a reply to Lin" ***Oxford Economic Paper,*** 2004

G. Laszlo, "Tax evasion, tax progression, and efficiency Wages" ***Economic Letters,*** 82, 2004

R.A. Musgrave, ***The Theory of Public Finance*** , 1959

A.T. Peacock and Show, G.K, "Tax Evasion and Tax Revenue Loss", ***Public Finance,*** 1982.

L.Ping, "Process and product R & D by a multiproduct monopolist" ***Oxford Economic Paper,*** 2004.

S. Watanabe, "Income Tax Evasion : A theoretical Analysis", ***Public Choice Studies*** No 8, 1986

S. Watanabe, "Tax Evasion and Asymmetric Laffer Curve", ***Public Choice Studies*** No 10.,1987.

S. Watanabe, "A Note on Taxation & Understated Monopoly Profit : Tax Evasion & Oversupply of Monopoly" ***Public Choice Studies*** No 14.1989.