

# Dynamic Characteristics of D-C Separately Excited Servomotor Driven by Thyratrons

著者	Ishizaki Takemitsu, Ishikawa Sadao
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# Dynamic Characteristics of D-C Separately Excited Servomotor Driven by Thyratrons

Takemitsu ISHIZAKI\* and Sadao ISHIKAWA\*

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This paper deals with starting and plugging characteristics of d-c separately excited servomotor driven by thyratrons. At steady state running, the output voltage of thyratrons which is supplied to armature of servomotor, greatly decreases with increase of its load current. There exists approximately a linear relation between its voltage and current. Therefore, d-c source supplied to motor by thyratrons, may be taken for the constant voltage source with a high internal resistance. By making use of this assumption, dynamic characteristics may be easily calculated and it can be verified that the time constant of d-c separately excited servomotor driven by thyratrons is greatly larger than by the ordinary d-c source such as the dynamo and the battery.

## 1. Introduction

In recent years, several papers<sup>1)2)</sup> have been published on the dynamic characteristics of d-c separately excited motor driven by thyratrons in automatic control systems. Although there are considerable informations for small variation of speed, very little work has been done to estimate the transient characteristics at large variation of speed such as starting and plugging.

In an earlier paper,<sup>3)</sup> the dynamic characteristics of d-c series motor driven by thyatron were reported. It was shown that the time constant of the series motor driven by thyatron is several times larger than that driven by the ordinary d-c source.

The present paper deals with the starting and plugging characteristics of d-c separately excited servomotor driven by thyratrons.

## 2. Circuit for thyatron control of d-c separately excited servomotor

The counter electromotive force  $E_c$ , induced in the armature of d-c separately excited servomotor under the constant field flux, is directly proportional to the speed  $N$ . Hence it can be expressed by

$$E_c = C_1 N \quad (1)$$

where  $C_1$  = coefficient of counter electromotive force

An equivalent circuit of a d-c separately excited servomotor is represented by a series circuit consisting of a resistance  $R$ , an inductance  $L$  in the armature winding and a battery  $E_c$ .

Fig. 1 shows the elementary circuit for thyatron control of a d-c separately excited servomotor. In Fig. 1,  $e_b$  and  $E_c$  represent, respectively, the specified grid bias voltage

\* Department of Electrical Engineering, College of Engineering.

and the signal voltage which serves for the control of the speed and direction of the servomotor, by controlling the ignition angle of the thyatron. The thyatron fires at the instant when the resultant grid voltage gets zero, neglecting the critical grid voltage. Therefore the angle of ignition  $x_f$  is given by

$$E_s = - [e_b]_{x=x_f} = B(1 + \cos x_f) \tag{2}$$

When  $E_s$  is positive, thyatron  $T_1$  conducts,  $T_2$  ceases conducting and the direction of rotation is positive; when  $E_s$  is negative,  $T_2$  conducts and the direction of rotation is reversed.

Fig. 2 shows a simplified circuit of thyatron motor system, in which a thyatron in no conduction is neglected. There, the servomotor is represented by an equivalent circuit ( $E_c - R - L$ ). When the motor is driven by thyatron,  $N$  and  $E_c$  have a positive sign; when the motor is braked by thyatron,  $N$  and  $E_c$  have a negative sign.

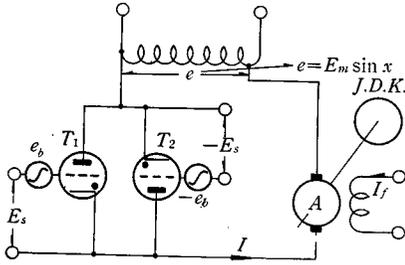


Fig. 1. Circuit for thyatron control of d-c separately excited servomotor.

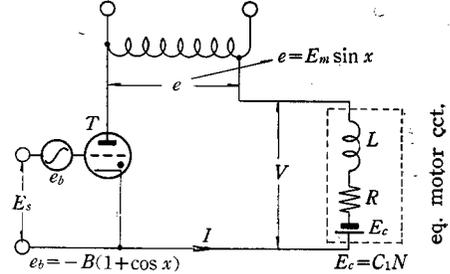


Fig. 2. Simplified circuit for thyatron control of d c separately excited servomotor.

### 3. Static characteristics

**3-1. Average values of voltage and current.** Fig. 3 represents the time functions of the anode supply voltage and of the load current flowing in the armature of a d-c servomotor. Referring to Fig. 3,  $E_0$  is the arc-voltage drop of the thyatron tube and  $x_e$  is the extinction angle. As the thyatron starts to conduct at positive anode voltage, it can be easily understood that the angle of ignition  $x_f$  is subject to very definite limitation;

$$x_1 < x_f < x_2$$

From Fig. 2, the general circuit equation can be written as following,

$$X \frac{di}{dt} + Ri = E_m \sin x - (E_0 + E_c) \tag{3}$$

where  $X = \omega L$

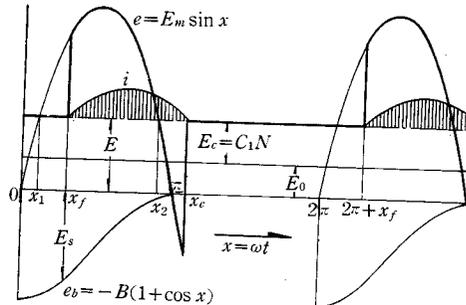


Fig. 3. Voltage and current waveforms of thyatron supplying to armature.

Since  $i=0$  at  $x=x_f$ , the current can be expressed as

$$i = \frac{E_m}{R} \left[ \{ \cos \theta \sin(x-\theta) - a \} - \{ \cos \theta \sin(x_f-\theta) - a \} e^{-\frac{x-x_f}{\tan \theta}} \right] \quad (4)$$

where  $\theta = \tan^{-1} \frac{X}{R}$ ,  $a = \frac{E}{E_m}$ ,  $E = E_0 + E_c$

As  $i=0$  at  $x=x_e$ , the relation between ignition angle  $x_f$  and extinction angle  $x_e$  is,

$$\{ a - \cos \theta \sin(x_e - \theta) \} e^{\frac{x_e}{\tan \theta}} = \{ a - \cos \theta \sin(x_f - \theta) \} e^{\frac{x_f}{\tan \theta}} \quad (5)$$

The solution for  $x_e$  of eq. (5) is represented by a family of graphs in Fig. 4, where graphs of  $x_e = f(x_f)$  are given for different values of  $a$  and  $\cos \theta$ .

The average value of the armature current  $I$  can be derived from eq. (4):

$$I = \frac{1}{2\pi} \int_{x_f}^{x_e} i dx = \frac{E_m}{2\pi R} [(\cos x_f - \cos x_e) - a(x_e - x_f)] \quad (6)$$

The average value  $V$  of the voltage at the armature terminals is equal to the sum of the counter electromotive force  $E_c$  and the armature voltage drop  $IR$ :

$$V = E_c + IR \quad (7)$$

Thus, from eqs. (6) and (7),

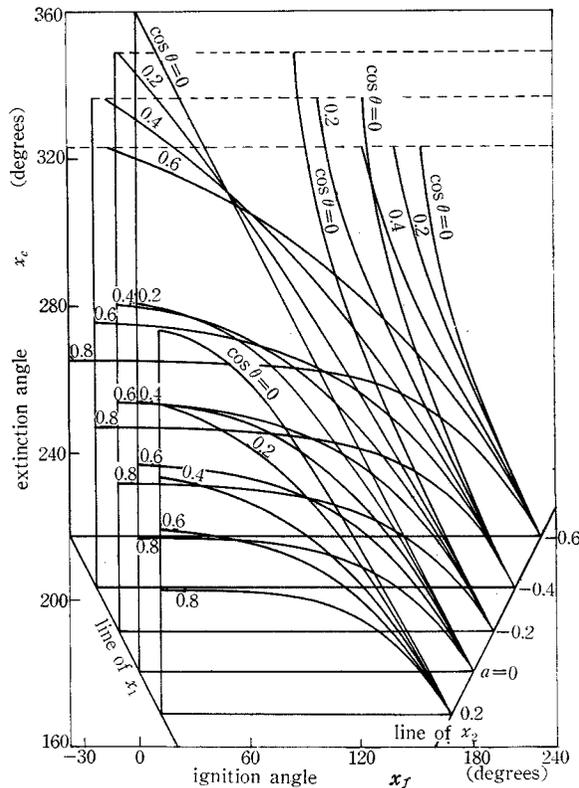


Fig. 4. Graphs of extinction angle vs. ignition angle for different parameters  $a$  and  $\cos \theta$ .

$$V = \frac{E_m}{2\pi} [(\cos x_f - \cos x_e) - a(x_e - x_f) + 2\pi a_0] \quad (8)$$

$$\text{where } a_0 = \frac{E_c}{E_m} = \frac{C_1}{E_m} N$$

**3-2. Static characteristics.** The characteristic curves of the thyatron motor system in the steady state running, can be plotted from the eqs. (6), (7) and (8). Fig. 5 and Fig. 6 show respectively the theoretical voltage-current and speed-current characteristics for  $x_f=90^\circ$ ,  $a_0=E_c/E_m=0.1$  and five different load impedance angle. In Figs. 5 and 6, dotted lines show border lines between the continuous and discontinuous conductions.

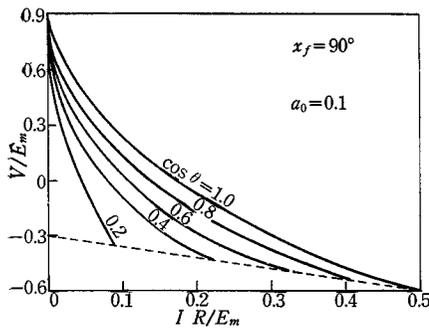


Fig. 5. Theoretical voltage vs. Current characteristics.

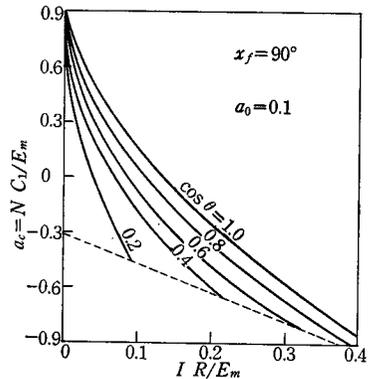


Fig. 6. Theoretical speed vs. Current characteristics.

Fig. 7 shows the diagram of the thyatron motor system in which a d-c machine load is used for testing. The speed  $N$  has a positive sign when the motor and its load are driven by thyatron;  $N$  has a negative sign when the motor is driven by d-c machine and it is braked by thyatron. The rating of the d-c separately excited servomotor with a tachometer generator is 27.5V, 0.8A, 10W and 3,500 rpm.

The experimental characteristics of thyatron motor system for  $x_f=90^\circ$  and  $E_m=70V$  are shown in Fig. 8. Curve I shows the speed  $N$ , curve II shows the average value of thyatron output voltage  $V$  which is supplied to armature.

The theoretical values for  $V$  are illustrated by the dotted line. Tolerable agreement between the experimental and theoretical curves is seen from Fig. 8. The curves I and

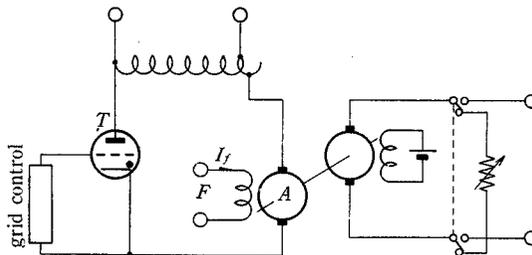


Fig. 7. Circuit for thyatron control of motor with load.

II, which show the speed-current and voltage-current characteristics respectively, have the discontinuous points at  $N=0$ , because of the influences due to coulomb friction and armature reaction.

In any case, whether  $N$  is positive or negative, voltage-current curve can be approximated by the straight line.

#### 4. Dynamic characteristics

**4-1. Equations for dynamic characteristics.** The output voltage of the thyatron circuit which is supplied to the armature of the servomotor, greatly decreases with the increase of its load current. There exists approximately a linear relation between its voltage and current. Therefore, the voltage  $V$  can be written as follows,

$$V = V_e - R_0 I \quad (9)$$

where  $V_e$  and  $R_0$  are constants which can be determined by the ignition angle, a-c supply voltage and impedance angle of armature circuit. From the eq. (9), the d-c source supplied by the thyatron to the armature of servomotor, may be taken for the d-c source of constant voltage  $V_e$  with a high internal resistance  $R_0$ .

From the eqs. (1), (7) and (9),  $V_e$  is represented as follows,

$$V_e = (R_0 + R)I + E_e = R_e I + C_1 N \quad (10)$$

$$\text{where } R_e = R_0 + R$$

Therefore,  $V_e$  and  $R_e$  can be called respectively apparent voltage and resistance in the thyatron motor system.

The torque developed in the d-c separately excited servomotor under constant field flux, is directly proportional to the armature current. Then the torque equation can be written:

$$\tau = C_2 I = J \frac{dN}{dt} + DN + K (\text{sgn}N) \quad (11)$$

where  $C_2$  = Coefficient of torque,

$J$  = Inertia of motor and load,

$D$  = Viscous friction coefficient of motor and load,

$K$  = Coulomb friction of motor and load.

**4-2. Dynamic characteristics.** The transient characteristics of the servomotor at starting and plugging can be calculated from eqs. (10) and (11).

At starting when constant signal voltage  $E_s$  is supplied to the grid circuit of the

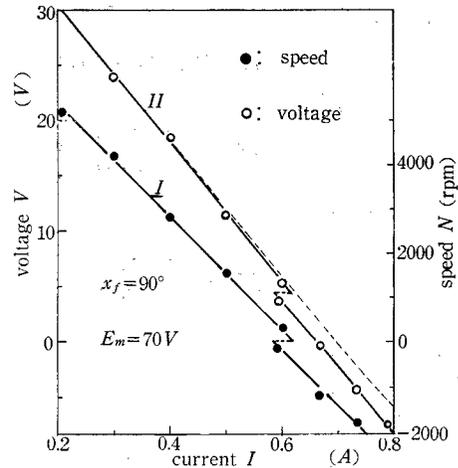


Fig. 8. Experimental voltage, speed vs. current characteristics.

thyatron motor system at rest, since  $N=0$  at  $t=0$ ,  $N$  is represented as follows,

$$N=N_{p\infty}(1-e^{-t/T_1}) \quad (12)$$

where

$$N_{p\infty}=\frac{C_2V_e-KR_e}{C_1C_2+DR_e}, \quad T_1=\frac{JR_e}{C_1C_2+DR_e} \quad (13)$$

For simplification, assuming that  $D$  is negligible, then,

$$N_{p\infty}=\frac{C_2V_e-KR_e}{C_1C_2}, \quad T_1=\frac{JR_e}{C_1C_2} \quad (14)$$

From eqs. (13) and (14), it is readily seen that the time constant  $T_1$  of motor is not a function of its own resistance  $R$  but a function of its apparent resistance  $R_e$ .

$R_e$  is several times larger than  $R$ . Therefore, the time constant of servomotor driven by the thyatron is larger than by ordinary d-c source such as the dynamo and the battery.

At plugging when the servomotor is running in one direction at the speed  $(-N_0)$  by the signal  $(-E_s)$  and is quickly reversed by the signal  $(+E_s)$ , since  $N=N_0$  at  $t=0$ ,

$$N=(N_{n\infty}+N_0)e^{-t/T_1}-N_{n\infty} \quad (15)$$

where

$$N_{n\infty}=\frac{C_2V_e+KR_e}{C_1C_2+DR_e} \quad (16)$$

At coasting when the signal is disconnected from the grid circuit of the thyatron motor system at speed  $N_0$ , allowing the motor to coast to a stop,  $N$  is represented as follows,

$$N=(N_{0\infty}+N_0)e^{-t/T_0}-N_{0\infty} \quad (17)$$

where

$$N_{0\infty}=K/D, \quad T_0=J/D \quad (18)$$

Neglecting the viscous friction,  $N$  can be written as

$$N=N_0-\frac{K}{J}t \quad (19)$$

From the above equation, the coasting characteristic is represented by a straight line.

**4-3. Experimental results.** The circuit employed for the dynamic test of the thyatron motor system, is shown in Fig. 1. The rating of the servomotor is 27.5V, 0.8 A, 10W and 3,500 rpm.

Fig. 9 shows the oscillograms of speed and current at reversing of motor (plugging and starting), and at coasting of motor. The tests were conducted on  $E_m=70V$ ,  $\alpha_f=90^\circ$ ,  $N_0=3,500$  rpm and  $I=0.18A$ .

In Fig. 9, the oscillogram of speed at coasting can be approximated by the straight line. Then the viscous friction nearly equals to zero.  $J/K$  is calculated from the average slope of speed oscillogram, experimentally. Therefore, the starting characteristic can be plotted from the eqs. (12) and (14).

The comparison between the experimental and calculated characteristics at starting is

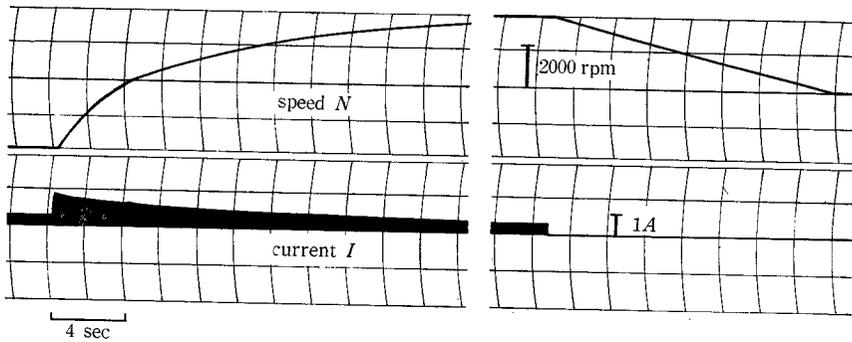


Fig. 9. Speed and current oscillograms.

shown in Fig. 10. Curve *I* shows the experimental result and the dotted line shows the calculated result. The agreement between both results is seen to be good. Curve *II* shows starting characteristic of the same servomotor driven by the ordinary d-c source, such as the dynamo and the battery. From curves *I* and *II*, it can be readily seen that the time constant of the servomotor controlled by the thyatron is several times larger than that driven by the ordinary d-c source.

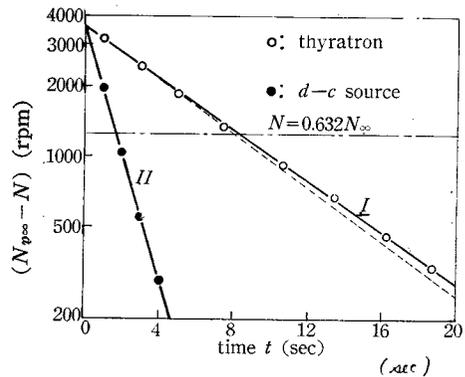


Fig. 10. Graphs of motor speed at starting.

### 5. Conclusions

At steady state of the thyatron motor system, the output voltage of the thyatron which is supplied to the armature of d-c separately excited servomotor, greatly decreases with the increase of its load current. There exists approximately a linear relation between its voltage and current. Therefore, the d-c source supplied to the armature of motor by the thyatron may be taken for the constant voltage source with a high internal resistance. By making use of this assumption, the dynamic characteristics of the thyatron motor system can be calculated easily.

The experimental results agree with the theoretical results fairly well. It can be theoretically verified that the time constant of the d-c separately excited servomotor driven by the thyratrons is greatly larger than that driven by the ordinary d-c source such as the dynamo and the battery.

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