<ARTICLE>Employment Policy For the Aged : Efficiency-Wage, Desired Wage and Pension

著者	Watanabe Shigeru, Kambashi Sonoko
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Employment Policy For the Aged**
— Efficiency-Wage, Desired Wage and Pension —

By Shigeru Watanabe †
and Sonoko Kambashi ‡

1 Introduction

Concerning social secuirity in Japan, pension system is one of the most important systems. Since the average life expectancy has become long, employment policy for the aged is required.

A purpose of this paper is to explore the employment policy considering a relationship between efficiency wage[1] and new pension system[2], which will be shortly described below.

In this paper, worker after the first-retirement will be mainly analyzed. New pension system has began in 1995. Under the new pension system, re-employed worker get additional income depending on the ratio between wage, w_a before retirement and that, w_b of re-employment.

In the next section a simple model will be examined. Concluding remarks will be given in the last section.

2 A Simple Model

2.1 Desired Wage Level of Re-Employed Worker

It is assumed that labor time per month is given and if wage income per month, w_b, of re-employed worker is low relative to that, w_a, before quitting a firm, he will get additional income, $\alpha(\rho)w_b$, depending on the ratio, ρ ($\equiv w_b/w_a$) and w_b.

The amount of pension, z, is assumed to be reduced to $z-\beta w_b$ when the

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†*Professor of Economics at the Osaka Prefecture University*, 1-1, *Gakuen-cho, Sakai, Osaka*, 599-8531, *Japan*

‡*Doctoral Student of Economics in the Osaka Prefecture University*

[1] *See Blanchard* (1989), *Stiglitz,J* (1976), *Summers,L.H.* (1988) *and Watanabe* (1996*a*, 1996*b*, 1997, 1999).

[2] *See Breyer* (1993), *Brunner* (1996), *Diamond,P.A.* (1965), *Feldstein,Martin* (1995) *and Kambashi* (1998, 1999*a*, 1999*b*).

worker is re-employed with the wage income, wb. More, it is assumed that according to the ratio, ρ , his pension will be further reduced to z- β wb- γ (ρ) × z, where β is constant and $\gamma(\rho)$ is a decreasing function of ρ. Hence the income of the re-employd laborer will be denoted by I;

$$I = w_b + \alpha(\rho)w_b + z - \beta w_b - \gamma(\rho)z. \tag{1}$$

To make the anarysis simple, the fuinctions are specified in the following manner:

$$\alpha(\rho) = \alpha_0(1 - \rho), \tag{2}$$

where $\alpha_0 > 0$, and $\rho = w_b / w_a$,

$$\gamma(\rho) = \gamma_0(1 - \rho), \tag{3}$$

where $\gamma_0 > 0$.
Therefore, equation (1) is rewritten as

$$\begin{aligned} I &= w_b + \alpha_0\left(1 - \frac{w_b}{w_a}\right)w_b + z - \beta w_b - \gamma_0\left(1 - \frac{w_b}{w_a}\right)z \\ &= \left\{1 + \alpha_0\left(1 - \frac{w_b}{w_a}\right) - \beta\right\}w_b + \left\{1 - \gamma_0\left(1 - \frac{w_b}{w_a}\right)\right\}z. \end{aligned} \tag{4}$$

Desired level of w_b for re-employed laborer can be obtained by maximizing (4)[3] with respect to w_b;

$$w_b^* = \frac{w_a}{2_{a0}}\left(1 + a_0 - \beta + \frac{\gamma_0 z}{w_a}\right). \tag{5}$$

Properties of w_b^* with respect to α_0, β, γ_0, w_a and z can straightforwardly be derived from (5);

$$\frac{\partial w_b^*}{\partial \alpha_0} = \frac{-2\{\gamma_0 z + (1 - \beta)w_a\}}{4\alpha_0^2} < 0, \tag{6}$$

[3] $\partial I / \partial w_b = -(w_b / w_a)\alpha_0 + \{1 + (1 - w_b / w_a)\alpha_0 - \beta\} + \gamma_0 z / w_a = 0,$
$\partial^2 I / \partial w_b^2 = -2\alpha_0 / w_a < 0.$

(7) $$\frac{\partial w_b^*}{\partial \beta} = \frac{-w_a}{2\alpha_0} < 0,$$

(8) $$\frac{\partial w_b^*}{\partial \gamma_0} = \frac{z}{2\alpha_0} > 0,$$

(9) $$\frac{\partial w_b^*}{\partial w_0} = \frac{1-\beta+\alpha_0}{2\alpha_0} > 0,$$

(10) $$\frac{\partial w_b^*}{\partial z} = \frac{\gamma_0}{2\alpha_0} > 0.$$

Hence, the effects of the changes in α_0, β, γ_0, w_a or z on the desired level of wage income for re-employed worker can be summarized in the following Table 1.

[Table 1]

	α_0	β	γ_0	w_a	z
w_b^*	[−]	[−]	[+]	[+]	[+]

As the results of examining the properties of w_b^*, we can get the following results; the higher the levels of γ_0, w_a, z (or the lower the level of α_0 and β), the higher the desired level of wage income of re-employed worker.

2.2 Efficiency-Wage and Pension

In this section the employment policy for the aged will be examined, taking efficiency-wage and pension into considerdation.

In the following, it is assumed that efficiency of worker depends on the difference between desired wage rate, w_b^* and actual wage rate, w_b which is equal to wage income per month.

Hence, the efficiency, e_b of re–employed worker can be simply specified as

(11) $$e_b = e_0 - e_1(w_b^* - w_b),$$

where $w_b^* > w_b$ is assumed.

In addition, we consider the employment policy in order to encourage re-employment of workers. It is assumed in the following that a part, θ of labor costs of re-employed worker is subsidized by the government for the re-employment of the aged.

Hence, the profit of the firm, π is shown as

(12) $$\pi = pf(e(\cdot)l_b) - w_b l_b.$$

Equation (12) is rewritten from (11) as,

(13) $$\pi = pk\log\{e_0 - e_1(w_b^* - w_b)l_b - (1 - \theta)w_b l_b.$$

where production function is specified as

(14) $$f(\cdot) = k\log\{e_0 - e_1(w_b^* - w_b)\}l_b,$$

where p is price level and considering a representative re-employed worker l_b can be reduced to one hereafter.

Maximizing π with respect to w_b yields the optimal wage rate, w_b^{**} for the firm;

(15) $$w_b^{**} = w_b^* + \frac{pk}{1-\theta} - \frac{e_0}{e_1},$$

where w_b^{**} is the desired wage rate shown by (5) for the re–employed worker.

Properties of w_b^{**} concerning $e_0, e_1, p, k, \alpha_0, \beta, \gamma_0, y_a, z$ or θ can be obtained straightforwardly;

(16) $$\frac{\partial w_b^{**}}{\partial e_0} = \frac{-1}{e_1} < 0,$$

(17) $$\frac{\partial w_b^{**}}{\partial e_1} = \frac{e_0}{e_1^2} > 0,$$

(18) $$\frac{\partial w_b^{**}}{\partial \theta} = \frac{pk}{(1-\theta)^2} > 0,$$

(19) $$\frac{\partial w_b^{**}}{\partial \alpha_0} = \frac{-(1-\beta)y_a + \gamma_0 z}{2\alpha_0} < 0,$$

(20) $$\frac{\partial w_b^{**}}{\partial \beta} = \frac{-y_a}{2\alpha_0}$$

(21) $$\frac{\partial w_b^{**}}{\partial w_a} = \frac{1-\beta+a_0}{2\alpha_0} > 0,$$

(22) $$\frac{\partial w_b^{**}}{\partial \gamma_0} = \frac{z}{2\alpha_0} > 0,$$

(23) $$\frac{\partial w_b^{**}}{\partial z} = \frac{\gamma_0}{2\alpha_0} > 0,$$

(24) $$\frac{\partial w_b^{**}}{\partial p} = \frac{k}{1-\theta} > 0,$$

(25) $$\frac{\partial w_b^{**}}{\partial k} = \frac{p}{1-\theta} > 0,$$

Therefore the above results can be summarized in the following Table 2.

[Table 2]

	e_0	e_1	θ	α_0	β	w_a	γ_0	z	p	k
w_b^{**}	[−]	[+]	[+]	[−]	[−]	[+]	[+]	[+]	[+]	[+]

Hence, we get the following results; for examples, (i) the optimal wage rate for the firm, w_b^{**} will be raised by the increase in the amount of the pension payment, z, (ii) w_b^{**} will be raised by a increase in the sensitivity, e_1 of efficiency with respect to wage rate, (iii) w_b^{**} will be raized by the increase in θ, which is a rate at which labor costs of re-employed worker is subsidized by the government for the re-employment of the aged, (iiii) w_b^{**} will be raised by the increase in w_a, which is the level of wage income per month before quitting a firm.

Next we examine the effect of raising θ on the output level, Q^{**}, which is derived from (5), (11), (14) and (15) as;

(26) $$Q^{**} = k\log\left\{\frac{e_1 pk}{1-\theta}\right\}.$$

Differentiating Q with respect to θ yields;

(27) $$\frac{\partial Q^{**}}{\partial \theta} = \frac{k}{1-\theta} > 0.$$

Hence, raising the subsidy rate, θ will increase the output level from (27) as well as the wage rate, w_b^{**} of re-employed worker from (18).

Net tax revenue is denoted by NTR^{**},

$$(28) \qquad NTR^{**} = t[pk\log\{e_0 - e_1(w_b^* - w_b^{**})\} - (1-\theta)w_b^{**}] - \theta w_b^{**}.$$

Differentiating NTR^{**} with repect to θ yields

$$(29) \qquad \frac{\partial NTR^{**}}{\partial \theta} = -(1-t)w_b^{**} - \frac{\theta pk}{(1-\theta)^2} < 0.$$

Hence raising the subsidy rate, θ, will decrease the net tax revenue.

On the other hand, total tax revenue TTR^{**}, which is composed of both net tax revenue and tax on labor income, is denoted by

$$(30) \qquad TTR^{**} = NTR^{**} + t_w w_b^{**}.$$

The effect of raising θ on TTR^{**} is not, however, determined in general. Since w_b^{**} is raised by the increase in θ from (18). Therefore, rasing θ for the re-employment of the aged will not necesaarilly decrease total tax revenue.

3 Concluding Remarks

In this paper, the employment policy for the aged is analyzed considering the relationship between the efficiency wage and new pension system.

Main results derived from the analysis can be summarized as follow: the policy of rasing subsidy rate, θ, for the re-employment of the aged will increase not only the wage of the re-employment worker but also the output level of the firm.

The effect of rasing the subsidy rate on net tax revenue; tax revenue from the profit of firm minus the amount of subsidy, is negative.

However, the total tax revenue which is composed of net tax revenue and tax on labor income is not necessarily decreased by the increase in the rate of subsidy rate for the re-employment of the aged.

The model, where labor employment is assumed to be given, i.e. one unit, can straightforwardly be generalized to allow for variable employment. In the generalized model, similar results can also be derived as shown in appendix; for example, the policy of raising θ for the aged will increase the output level.

However, in the generalized model, the policy of raising θ will increase the employment level of the aged, while the policy will have no effect on w, as derived in appendix. Other intensive analysis by using generalized models including the model developed in appendix will be made in forthcoming paper.

In this paper reservation wage is implicitly assumed to be sufficiently low and the efficiency of the worker is assumed to depend on the difference between the desired wage and actual wage per month. Further in appendix, two workers with different desired wages are analyzed. As the result of the analysis it is derived that the wage which maximizes the profit may be higher or lower than the desired wages depending on the level of pension.

In this paper only the re-employed worker is analyzed. However, the relationship between the employment of the re-employed workers and that of young workers will be important. The efficiency of young workers may depend not only the level of wage but also the probability of re-employment. It will also be important to examine the effect of the policy for the aged not only on the employment of the aged but also on that of young workers. These issues will be analyzed intensively in forthcomming paper. As tentatively examined in appendix, for example, rasing the subsidy rate θ for the wage payment to the aged will have negative effect on the wage of young workers, while it has no effect on the wage of the aged.

Appendix

(A − 1) $$\pi = pk\log\{e_0 - e_1(w_b^* - w_b)^2\}l_b - (1-\theta)w_b l_b,$$

where e is specified as $e = e_0 - e_1(w_b^* - w_b)^2$.

From the conditions which maximize π denoted by (A − 1) with respect to w_b and l_b, we obtain w_b^{**} and l_b^{**};

(A − 2) $$w_b^{**} = \left[\frac{\{1+\alpha_0-\beta)w_a+\gamma_0 z\}^2}{4\alpha_0^2} - \frac{e_0}{e_1}\right]^{1/2},$$

and

(A − 3) $$l_b^{**} = \frac{pk}{(1-\theta)}\left[\frac{\{1+\alpha_0-\beta)w_a+\gamma_0 z\}^2}{4\alpha_0^2} - \frac{e_0}{e_1}\right]^{-1/2},$$

where in general l_b^{**} is, needless to say, not equal to actual supply of labor at $w_b = w_b^{**}$.

It can straghtforwardly be derived that second-order conditons are satisfied if θ, $0 < \theta < 1$, is sufficiently high.

From (A − 2) and (A − 3), following results can directly be obtained;

$$\text{(A − 4)} \qquad \frac{\partial l_b^{**}}{\partial \theta} = 0,$$

$$\text{(A − 5)} \qquad \frac{\partial w_b^{**}}{\partial \theta} = 0$$

and

$$\text{(A − 6)} \qquad \frac{\partial Q^{**}}{\partial \theta} > 0,$$

where Q^{**} is defined by

$$\text{(A − 7)} \qquad Q^{**} = k\log\{e_0 - e_1(w_b^{*} - w_b^{**})^2\}l_b^{**}.$$

Considering two workers with $z = z_1$ or z_2, π is shown as;

$$\text{(A − 8)} \qquad \pi = pk\log[\{e_0 - e_1(w_{b1}^{*} - w_b)^2\}l_{b1} + \{e_0 - e_1(w_{b2}^{*} - w_b)^2\}l_{b2}] - (1 - \theta)(l_{b1} + l_{b2})w_b,$$

where w_{b1}^{*} or w_{b2}^{*} are the desired wages of the re-employed workers with $z = z_1$ or $z = z_2$ respectively, l_{b1} and l_{b2} are employment of the them and w_b is wage payed to them.

Differrentiating π with respect to w_b, l_{b1} and l_{b2} yields the first-order conditions;

$$\text{(A − 9)} \qquad \frac{\partial \pi}{\partial w_b} = pk\frac{2e_1(w_{b1}^{*} - w_b)l_{b1} + (w_{b2}^{*} - w_b)l_{b2}\}}{\{e_0 - e_1(w_{b1}^{*} - w_b)^2\}l_{b1} + \{e_0 - e_1(w_{b2}^{*} - w_b)^2\}l_{b2}} - (1 - \theta)(l_{b1} + l_{b2}) = 0,$$

(A − 10) $$\frac{\partial \pi}{\partial l_{b1}} = pk \frac{e_0 - e_1(w_{b1}^* - w_b)^2}{\{e_0 - e_1(w_{b1}^* - w_b)^2\}l_{b1} + \{e_0 - e_1(w_{b1}^* - w_b)^2\}l_{b2}}$$

$$-(1-\theta)w_b$$

$$= 0,$$

(A − 11) $$\frac{\partial \pi}{\partial l_{b2}} = pk \frac{e_0 - e_1(w_{b2}^* - w_b)^2}{\{e_0 - e_1(w_{b1}^* - w_b)^2\}l_{b1} + \{e_0 - e_1(w_{b2}^* - w_b)^2\}l_{b2}}$$

$$-(1-\theta)w_b$$

$$= 0.$$

Second-order conditions are assumed to be satisfied.
From (A − 9),

(A − 12) $$\frac{2e_1 pk}{lb_1 + lb_2}\{(w_{b1}^* - w_b)l_{b1} + (w_{b2}^* - w_b)l_{b2}\}$$

$$= (1-\theta)\left[\{e_0 - e_1(w_{b1}^* - w_b)^2\}l_{b1} + \{e_0 - e_1(w_{b2}^* - w_b)^2\}l_{b2}\right].$$

From (A − 10),

(A − 13) $$pk\{e_0 - e_1(w_{b1}^* - w_b)^2\}$$

$$= (1-\theta)w_b[\{e_0 - e_1(w_{b1}^* - w_b)^2\}l_{b1} + \{e_0 - e_1(w_{b2}^* - w_b)^2\}l_{b2}].$$

From (A − 11),

(A − 14) $$pk\{e_0 - e_1(w_{b2}^* - w_b)^2\}$$

$$= (1-\theta)w_b[\{e_0 - e_1(w_{b1}^* - w_b)^2\}l_{b1} + \{e_0 - e_1(w_{b2}^* - w_b)^2\}l_{b2}].$$

Substituting (A − 12) into (A − 13) and (A − 14) yields

$$pk\{e_0 - e_1(w_{b1}^* - w_b)^2\} = \frac{2e_1 pkw_b}{l_{b1} + l_{b2}}\{(w_{b1}^* - w_b)l_{b1} + (w_{b2}^* - w_b)l_{b2}\},$$

$$pk\{e_0 - e_1(w_{b2}^* - w_b)^2\} = \frac{2e_1pkw_b}{l_{b1} + l_{b2}}\{(w_{b1}^* - w_b)l_{b1} + (w_{b2}^* - w_b)l_{b2}\}.$$

Hence, from above equations we get

$$(w_{b1}^* - w_b)^2 = (w_{b2}^* - w_b)^2.$$

Therefore, as $w_{b1}^* \neq w_{b2}^*$, we obtain

$$w_b^{**} = \frac{1}{2}(w_{b1}^* + w_{b2}^*),$$

where w_b^{**} is wage which maximizes the profit of the firm.

Without loss of generality, suppose that $z_1 > z_2$, which implicates $w_{b1}^* > w_{b2}^*$. Then it is derived that $w_b^{**} < w_{b1}^*$ and $w_b^{**} > w_{b2}^*$.

Next, we suppose that

$$\text{(A - 15)} \quad \pi = pk\log[(r_0 + r_1w_a + r_2q)l_a + \{e_0 - e_1(w_b^* - w_b)^2\}l_b]$$
$$- w_al_a - (1 - \theta)w_bl_b,$$

where w_a is wage for young workers, q is the probability of re-employment which is given by $q = l_b/l_a$, l_a is the employment of young workers and the efficiency of young workers, e_y, is assumed to depend on w_a and q such that $e_y = r_0 + r_1w_a + r_2q$, $r_0 > 0$, $r_1 > 0$, $r_2 > 0$.

Maximizing π with respect to w_a, w_b, l_a and l_b yields

$$\text{(A - 16)} \quad \frac{\partial\pi}{\partial w_a} = pk\frac{r_1l_a - e_1(wb^* - wb)(1 + \alpha_0)l_b/\alpha_0}{(r_0 + r_1w_a)l_a + r_2l_b + \{e_0 - e_1(w_b^* - w_b)^2\}l_b} - l_a = 0,$$

$$\text{(A - 17)} \quad \frac{\partial\pi}{\partial w_b} = pk\frac{2e_1(w_b^* - w_b)l_b}{(r_0 + r_1w_a)l_a + r_2l_b + \{e_0 - e_1(w_b^* - w_b)^2\}l_b} - (1 - \theta)l_b = 0,$$

$$\text{(A - 18)} \quad \frac{\partial\pi}{\partial l_a} = pk\frac{r_0 + r_1w_a}{(r_0 + r_1w_a)l_a + r_2l_b + \{e_0 - e_1(w_b^* - w_b)^2\}l_b} - w_a = 0,$$

$$\text{(A - 19)} \quad \frac{\partial\pi}{\partial l_b} = pk\frac{r_2 + \{e_0 - e_1(w_b^* - w_b)^2\}}{(r_0 + r_1w_a)l_a + r_2l_b + \{e_0 - e_1(w_b^* - w_b)^2\}l_b} - (1 - \theta)w_b = 0.$$

Second-order conditons are assumed to be satisfied.
From (A − 17) we have

$$(r_0 + r_1 w_a) l_a + r_2 l_b + \{e_0 - e_1 (w_b^* - w_b)^2\} l_b = \frac{2e_1 pk}{1-\theta}(w^{b*} - w_b)$$

Substituting the above equation into (A − 19) yields

$$w_b^{*2} - w_b^{**2} = \frac{e_0 + r_2}{e_1} > 0,$$

then

$$w_b^{**} = \left(w_b^{*2} - \frac{e_0 + r_2}{e_1}\right)^{1/2},$$

where w_b^* is given by equation (5).
Substituting w_b^{**} into (A − 18) yields

$$w_a^{**} = \frac{r_0}{2e_1(w_b^* - w_b^{**})/(1-\theta) - r_1},$$

where $w_b^* > w_b^{**}$ as derived above. Hence we get

$$w_a^{**} = \frac{r_0}{2e_1[w_b^* - w_b^{*2} - (e_0 + r_2)/e_1\}^{1/2}]/(1-\theta) - r_1}.$$

Therefore we obtain

$$\frac{\partial w_a^{**}}{\partial \theta} < 0,$$

and

$$\frac{\partial w_b^{**}}{\partial \theta} = 0.$$

REFERENCES

Blanchard,O.J., and S.Fischer (1989), *Lectures on Macroeconomics*, M.I.T.Pr.,U.S.

Breyer,Friedrich and Straub,Martin (1993), "Welfare Effects of Unfunded Pension Systems when Labor Supply is Endogenous," *Journal of Public Economics*, Vol.50, pp.77 – 91.

Brunner,J.K. (1996), "Transition from a pay-as-you-go to a Fully Funded Pension Systems: The Case of Differing Individuals and Intragnerational Fairness," *Journal of Public Economics*, Vol.60, pp.131 – 146.

Diamond,P.A. (1965), "National Debt in a Neoclassical Growth Model," *American Economic Review*, Vol.55, pp.1126 – 50.

Feldstein,Martin (1995), "Would Privatizing Social Security Rise Economic Welfare?" *NBER Working Paper* 5281.

Stiglitz,J. (1976), "The Efficiency Wage Hypothesis, Surplus Labor, and the Distribution of Income in L.D.C.s," *Oxford Economic Papers*, Vol.28, July, pp.185 – 207.

Summers,L.H. (1988), "Relative Wages, Efficiency Wage and Keynesian Unemployment," *American Economic Review*, Vol.78, May, pp.383 – 388.

The Ministry of Welfare (1997), *Pension and Finance*, Houken.

The Ministry of Welfare (1997), *Introduction to Social Security*, Cyuohoki – Syuppan.

Sigeru Watanabe (1996a), "A Note on a Basic Model of Efficiency Wage and Tax Evasion," *Bulletin of Osaka Prefecture University* Series D, Vol.XL, pp.43 – 47.

——————(1996b), "Further Note on Efficiency Wage & Tax Evasion," *Economic Studies*, (Osaka Prefecture University) Vol.41, No3, pp.49 – 62.

——————(1997), "Tax Evasion, Product Liability and Efficiency Wage," *Economic Studies*, (Osaka Prefecture University) Vol.42, No4, pp.1 – 12.

——————(1999), "Two Recently Important Issues — "In-efficiency" -Wage Hypothesis and Entertainment Expense & Tax Evasion," *Economic Studies*, (Osaka Prefecture University) Vol.44, No3, pp.1 – 10.

Sonoko Kambashi (1998), "Privatizing of Public Pensions," *Shirasagi Ronso,* (Osaka Prefecture University) Vol.30, pp.1 – 20.

——————————(1999a), "Overlappng Generations Models and Public Pensions," *Economic Studies,* (Osaka Prefecture University) Vol.45, No.1, pp.113 – 134.

——————————(1999b), "The Economic Effects of Public Pension System when the Labor Supply is Endogenous," *Shirasagi Ronso,* (Osaka Prefecture University) Vol.31, pp.1 – 13.